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Symmetries of Julia Sets of Nondegenerate Polynomial Skew Products on $\mathbf{C}^2$

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1. Introduction

The Julia sets of any kind of functions or maps can have symmetries. We say that a Julia set has symmetries if some nonelementary transformations preserve it. Beardon [B] investigated the symmetries of the Julia sets of polynomials on $\mathbf{C}$. For the Julia set of a polynomial, the symmetries of the Julia set are rotations about some point. The group of symmetries is infinite if and only if the Julia set is a circle, which is equivalent to the polynomial being conjugate to $z \to z^d$. There was a problem for polynomials that had the same Julia set. Beardon [B] gave an answer to this problem in terms of a functional equation in which the symmetries of the Julia set are used. Finally, the problem was solved by [SS] and [AHu] independently: polynomials having the same Julia set are essentially the same.

We want to extend these dynamical objects and results in one dimension to those in higher dimensions. As a first step, we extend these dynamical objects and results of polynomials to those of nondegenerate polynomial skew products. Although the dynamics of polynomial skew products is a complicated dynamics in higher dimensions, it has many analogies to the dynamics of polynomials.

The paper is organized as follows. In Section 2, we recall the dynamics of a nondegenerate polynomial skew product and show the existence of the vertical Green functions and Böttcher functions of the map. In Section 3 we investigate the symmetries of the Julia set of a nondegenerate polynomial skew product. We show that suitable transformations preserving the Julia set are conjugate to rotational product maps, and we give a necessary and sufficient condition for the group of symmetries to be infinite. In Section 4 we deal with the problem case of nondegenerate polynomial skew products that have the same Julia set. We place a restriction on nondegenerate polynomial skew products and show that, except for two types, these maps having the same Julia set are essentially the same. The paper concludes with application of this result to the dynamics of regular polynomial skew products.

2. Dynamics of Polynomial Skew Products

Let us recall the dynamics of nondegenerate polynomial skew products on $\mathbf{C}^2$. Heinemann [H] and Jonsson [J] studied the dynamics of regular polynomial skew

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