Michigan Math. J. 58 (2009)

Global Solutions of Homogeneous Linear Partial Differential Equations of the Second Order

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1. Introduction and Main Results

In 1995, we proved in [9] that the meromorphic solutions of the system of partial differential equations

$$\frac{\partial u}{\partial z_j} = a_j(z) + b_j(z)u + c_j(z)u^2, \quad j = 1, 2, \ldots, m$$

(where a_j, b_j, c_j are polynomials on $\mathbb{C}^m$) are of finite positive order and are pseudo-prime. Li and Saleeby [13] characterized entire solutions in $\mathbb{C}^m$ of first-order partial differential equations of the form

$$\frac{\partial u}{\partial z_j} = f_j(u), \quad j = 1, 2, \ldots, m,$$

where the f_j are meromorphic functions in $\mathbb{C}$. Berenstein and Li [2] studied entire solutions in $\mathbb{C}^m$ for first-order partial differential equations of the form

$$\frac{\partial u}{\partial z_j} = p(z) f(u), \quad j = 1, 2, \ldots, m,$$

where p and f are entire or meromorphic functions in $\mathbb{C}^m$ and $\mathbb{C}$, respectively. Li [12] also gave a complete description of entire solutions of the Fermat type partial differential equation

$$\left(\frac{\partial u}{\partial z_1}\right)^m + \left(\frac{\partial u}{\partial z_2}\right)^n = 1.$$

In this paper, we study meromorphic solutions of homogeneous linear partial differential equations of the second order in two independent complex variables,

$$a_0 \frac{\partial^2 u}{\partial t^2} + 2a_1 \frac{\partial^2 u}{\partial t \partial z} + a_2 \frac{\partial^2 u}{\partial z^2} + a_3 \frac{\partial u}{\partial t} + a_4 \frac{\partial u}{\partial z} + a_6 u = 0; \tag{1}$$

here $a_k = a_k(t, z)$ are holomorphic functions for $(t, z) \in \Sigma$, where Σ is a region on $\mathbb{C}^2$.

Received August 26, 2008. Revision received November 17, 2008.
The work of the first author was partially supported by the Natural Science Foundation of China. Work of the second author was partially supported by a UGC Grant of Hong Kong, Project no. 604106.