Michigan Math. J. 58 (2009)

Proper Holomorphic Mappings between Reinhardt Domains in $\mathbb{C}^2$

ŁUKASZ KOSIŃSKI

1. Introduction

Given $\alpha \in \mathbb{R}^n$ and $z \in \mathbb{C}^n_*$, we put $|z^\alpha| := |z_1|^{\alpha_1} \cdots |z_n|^{\alpha_n}$ whenever it makes sense. Let $\mathbb{A}_{r^-,r^+} = \{z \in \mathbb{C} \mid r^- < |z| < r^+\}$ for $-\infty < r^- < r^+ < \infty$, $r^+ > 0$. By $\mathbb{D}$ we always denote the unit disc in $\mathbb{C}$. For a domain $D \subset \mathbb{C}^n$, $D \setminus \{0\}$ is denoted by D_*.

Following [Z2], for $A = (A^j_k)_{j=1,\ldots,m,\, k=1,\ldots,n} \in \mathbb{Z}^{m\times n}$ and $b = (b_1, \ldots, b_m) \in \mathbb{C}^m_*$ we define:

$$\varphi_A(z) := z^A := (z^{A^1}, \ldots, z^{A^m}), \quad z \in \mathbb{C}^n_*,$$

$$\varphi_{A,b}(z) := (b_1 z^{A^1}, \ldots, b_m z^{A^m}), \quad z \in \mathbb{C}^n_*.$$

Such maps are called *elementary algebraic* (or briefly *elementary* maps).

The aim of this paper is to describe nonelementary proper holomorphic maps between nonhyperbolic Reinhardt domains in $\mathbb{C}^2$ as well as the corresponding pairs of domains.

Recall that if D, G are Reinhardt domains and $f\colon D \to G$ is a biholomorphic mapping, then f can be represented as composition of automorphism of D and G and an elementary mapping between these domains (see [K] and [S2]). Thus, the description of nonelementary biholomorphic mappings between Reinhardt domains reduces to the investigation of their group of automorphisms. It is a general problem of complex geometry of Reinhardt domains considered in many papers. In [S1] the author used group-theoretic methods to investigate the holomorphic equivalence of bounded Reinhardt domains in $\mathbb{C}^n$ not containing the origin and thereby determined automorphisms of a certain class of Reinhardt domains. Similar results were obtained by Barrett in [Ba], although his approach was analytic. The groups of automorphisms of all bounded Reinhardt domains containing the origin were determined in [Su]. This work has been extended in [K] by dropping the assumption that the origin is included in the domain. The situation when domains D and G may be unbounded were considered for example in [S3] and [EZ].

Obviously, the problem of describing proper holomorphic mappings is harder to deal with. Proper maps between nonhyperbolic, pseudoconvex Reinhardt domains have been considered in [EZ] and [Ko]. In the bounded case, partial results

Received May 15, 2008. Revision received February 9, 2009.
Research partially supported by the KBN Grant no. N201 271435.