Michigan Math. J. 58 (2009)

On Local Models with Special Parahoric Level Structure

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0. Introduction

Motivation and Main Results

For the study of arithmetic properties of a variety over an algebraic number field, it is of interest to have a model over the ring of integers. In the particular case of a Shimura variety, one likes to have a model over the ring of integers $\mathcal{O}_E$, where E is the completion of the reflex field at a finite prime of residue characteristic p. It should be flat and have only mild singularities. If the Shimura variety is the moduli space over Spec E of abelian varieties with additional polarization, endomorphisms, and level structure (a Shimura variety of PEL type), then it is natural to define a model by posing the moduli problem over $\mathcal{O}_E$. In the case of a parahoric level structure at p with the parahoric defined in an elementary way as the stabilizer of a self-dual periodic lattice chain, such a model has been given by Rapoport and Zink [RZ].

Although in special cases this model is shown to be flat with reduced special fiber and with irreducible components that are normal and have only rational singularities [Gö1; Gö2], in general it is not flat, as has been pointed out by Pappas [P]. In a series of papers, Pappas and Rapoport [PR1; PR2; PR4] examine how to define closed subschemes of this *naive model* that are more likely to be flat. Flatness can be brutally enforced by taking the (reduced) Zariski closure of the generic fiber in the naive model. Aside from that, by adding further conditions one can attempt to cut out this closed subscheme, or at least give a better approximation. If the parahoric subgroup is the stabilizer of a self-dual periodic lattice chain, then these questions can be reduced to problems of the corresponding *local* models [RZ]. Locally for the étale topology around each point of the special fiber, these models coincide with the corresponding moduli schemes. This approach has the advantage of leading to varieties that can be defined in terms of linear algebra and thus can be handled more easily. In the setting of unitary groups considered here, Pappas [P] defines in this way the *wedge local model,* a closed subscheme of the naive local model. The *local model* is defined to be the closure of the generic fiber in the naive local model; it is also a closed subscheme of the wedge local model.

Received April 14, 2008. Revision received August 18, 2009.