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A Generalization of the Griffiths' Theorem on Rational Integrals, II

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0. Introduction

Let $X = \mathbf{P}^n$, and let $Y \subset X$ be a hypersurface defined by a reduced polynomial f of degree d. Set $U = X \setminus Y$. Let F and P denote, respectively, the global Hodge and pole order filtrations on the cohomology $H^n(U, \mathbf{C})$ (see [5; 6]). Locally it is easy to calculate the difference between these two filtrations at least in the case of isolated weighted homogeneous singularities; see (1.3.2) in the next section. However, this is quite nontrivial globally (i.e., on the cohomology). It is important to know when the two filtrations coincide globally, since the Hodge filtration and especially the Kodaira–Spencer map can be calculated rather easily if they coincide (see [9, Thm. 4.5]). It is known that they are different if Y has bad singularities (see [7] and also [9, 2.5]). In case the singularities consist of ordinary double points, however, it was unclear whether they still differ globally. They coincide for $n = 2$ in this case [7; 9], but the calculation for the case $n > 2$ is quite complicated in general. In this paper we prove the following result.

Theorem 1. *Assume $d = 3$ with $n \geq 5$ or $d = 4$ with $n \geq 3$. Set $m = [n/2]$, and assume that $1 + (n+1)/d \leq p \leq n - m$. Then, for a sufficiently general singular hypersurface Y, we have $F^p \neq P^p$ on $H^n(U, \mathbf{C})$.*

Here a *sufficiently general* singular hypersurface is one that corresponds to a point of a certain (sufficiently small) nonempty Zariski-open subset of $D \setminus \operatorname{Sing} D$, where D is the parameter space of singular hypersurfaces of degree d in $\mathbf{P}^n$; see Section 3.6. In particular, $\operatorname{Sing} Y$ consists of one ordinary double point. It is unclear whether the two filtrations differ whenever $\operatorname{Sing} Y$ consists of one ordinary double point. According to Theorem 1, the formula for the Kodaira–Spencer map in [9, Thm. 4.5] is effective only for $p > n - m$ in the ordinary double point case. By Theorem 2, however, we can show a similar formula in the ordinary point case that is valid also for $p \leq n - m$; see Corollary 4.5. In the case of n odd, we can also use the self-duality for the calculation of the Kodaira–Spencer map; see Remark 3.9(ii).

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