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# Holomorphic Motions, Fatou Linearization, and Quasiconformal Rigidity for Parabolic Germs

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## 1. Introduction

One of the fundamental theorems in complex dynamical systems is a theorem called the Fatou linearization theorem. This theorem provides a topological and dynamical structure of a parabolic germ. A parabolic germ $f$ is an analytic function defined in a neighborhood of a point $z_0$ in the complex plane $\mathbb{C}$ such that (a) it fixes $z_0$ and (b) some power $(f'(z_0))^q$ of the derivative $f'(z_0)$ of $f$ at $z_0$ is 1. Thus we can write $f(z)$ in the following form:

$$f(z) = z_0 + \lambda(z - z_0) + a_2(z - z_0)^2 + \cdots + a_n(z - z_0)^n + \cdots, \quad z \in U,$$

where $U$ is a neighborhood of $z_0$ and $\lambda = e^{2\pi pi/q}$ for $p$ and $q$ two relatively prime integers. The number $\lambda$ is called the *multiplier* of $f$. Two parabolic germs $f$ and $g$ at two points $z_0$ and $z_1$ are said to be *topologically conjugate* if there is a homeomorphism $h$ from a neighborhood of $z_0$ onto a neighborhood of $z_1$ such that

$$h \circ f = g \circ h.$$

If $h$ is a $K$-quasiconformal homeomorphism, then we say that $f$ and $g$ are *$K$-quasiconformally conjugate*.

By a linear conjugacy $\phi(z) = z - z_0$, we may assume that $z_0 = 0$. So we only consider parabolic germs at 0,

$$f(z) = \lambda z + a_2 z^2 + \cdots + a_n z^n + \cdots, \quad z \in U.$$

Assume we are given a parabolic germ $f$ at 0 whose multiplier $\lambda = e^{2\pi ip/q}$ with $(p, q) = 1$. Then

$$f^q(z) = z + az^{n+1} + o(z^{n+1}), \quad n \geq 1.$$

If $a \neq 0$, then $n + 1$ is called the *multiplicity* of $f$. Here $n = kq$ is a multiplier of $q$. The Leau–Fatou flower theorem states that the local topological and dynamical picture of $f$ around 0 can be described as follows. There are $n$ petals pairwise tangential at 0 such that each petal is mapped into the $(kp)$th petal counting counterclockwise from this petal. These petals are called *attracting* petals. At

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