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An Explicit $\bar{\partial}$-Integration Formula for Weighted Homogeneous Varieties

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1. Introduction

As is well known, solving the $\bar{\partial}$-equation forms a main part of complex analysis; but it also has deep consequences for algebraic geometry, partial differential equations, and other areas. In general, it is not easy to solve the $\bar{\partial}$-equation. The existence of solutions depends mainly on the geometry of the variety on which the equation is considered. There is a vast literature about this subject on smooth manifolds, both in books and papers (see e.g. [10; 11; 12]), but the theory on singular varieties has been developed only recently.

Let Σ be a singular subvariety of the space $\mathbb{C}^n$, and let λ be a bounded $\bar{\partial}$-closed differential form on the regular part of Σ. Fornæss, Gavosto, and Ruppenthal have proposed a general technique for solving the $\bar{\partial}$-equation $\lambda = \bar{\partial}g$ on the regular part of Σ, a technique they have successfully applied to varieties of the form $\{z^m = w_1^{k_1} \cdots w_n^{k_n}\} \subset \mathbb{C}^{n+1}$; see [6; 9; 14]. They exploit the fact that such a variety can be considered as an m-sheeted analytic covering of the complex space $\mathbb{C}^n$. The $\bar{\partial}$-equation is then projected into $\mathbb{C}^n$ by the use of symmetric combinations, and it is solved there with certain weights. The form g is constructed from the pull-back of a finite set of previous solutions. There is a certain chance for this strategy to work in general varieties, because any locally irreducible complex space can be locally represented as a finitely sheeted analytic covering over a complex number space.

On the other hand, Acosta, Solís, and Zeron have developed an alternative technique for solving the $\bar{\partial}$-equation (if λ is bounded) on any kind of singular quotient variety embedded in $\mathbb{C}^m$ and generated by a finite group of unitary matrices, such as hypersurfaces in $\mathbb{C}^3$ with only a rational double point singularity; see [1; 2; 18]. They use the quotient structure in order to pull back the $\bar{\partial}$-equation into a complex space $\mathbb{C}^n$ and to solve the original equation by using symmetric combinations. This strategy has the drawback that not all varieties are quotient ones.

In both of these approaches, the main strategy is to transfer the problem into some nonsingular complex space, to solve the $\bar{\partial}$-equation in this well-known situation, and then to carry over the solution into the singular variety Σ. The main objective

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