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Block Source Algebras in p-Solvable Groups

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Dedicated to the memory of Donald G. Higman

1. Introduction

1.1. The purpose of this paper is to fill a gap that has remained open since 1979, when in the Santa Cruz conference we announced the main results on the so-called local structure of the blocks of finite p-solvable groups [6], which were mainly obtained from a suitable translation to algebras of Fong's reduction [4]. At that time, the term *local structure* referred to the paper by Alperin and Broué [2], but since that meeting it has become clear that, when studying a block of a finite group, the structure to describe is its *source algebra*.

1.2. As a matter of fact, in [6] we already described the source algebra of a *nilpotent block* in a finite p-solvable group, and one of the reasons for delaying the publication of our work on the blocks of p-solvable groups was that, after Santa Cruz, we concentrated our effort on determining the structure of the source algebra of nilpotent blocks in *any* finite group [10].

1.3. Another reason for delaying this publication was that, although the translation to algebras of Fong's reduction does indeed allow one to determine the structure of the source algebra of a block in finite p-solvable groups, this structure involves a *Dade P-algebra,* where P is a defect p-subgroup of the block, and only many years later did we find a way to prove its uniqueness. A last remark: although, for the sake of simplicity, we deal only with the source algebra of a block in characteristic $p > 0$, the interested reader will see that [10, Lemma 7.8] and [11, Cor. 3.7] allow one to determine the source algebra over a complete discrete valuation ring of characteristic 0.

2. Notation and Quoted Results

2.1. We fix a prime number p and an algebraically closed field of characteristic p. It is well known that Fong's reduction involves a central extension of the finite group we start with; precisely, it involves a central extension by a finite subgroup of k^*, and a handy way to unify our setting is to consider from the beginning a central extension $\hat{G}$ of a finite group G by k^*. This is not more general since, nevertheless, $\hat{G}$ always contains a *finite* subgroup G' covering G.

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