Michigan Math. J. 56 (2008)

A Sharp Bound for the Slope of Double Cover Fibrations

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0. Introduction and Preliminaries

A *fibered surface,* or simply a *fibration,* is a proper surjective morphism with connected fibers f from a smooth surface X to a smooth complete curve B. Call F the general fiber of f. A fibration is said to be *relatively minimal* if the fibers contain no (-1)-curves. The genus g of F is called the *genus of the fibration.* We say that f is smooth if all the fibers are smooth, isotrivial if all the smooth fibers are mutually isomorphic, and *locally trivial* if it is smooth and isotrivial. A *hyperelliptic* (resp. *bielliptic*) *fibration* is a fibered surface whose general fiber is a hyperelliptic (resp. bielliptic) curve. A fibration is said to be *semistable* if all its fibers are reduced nodal curves that are moduli semistable (any rational smooth component meets the rest of the curve in at least two points).

Relative Invariants. As usual, the *relative dualizing sheaf* of a fibration $f: X \to B$ is the line bundle

$$\omega_f = \omega_X \otimes (f^*\omega_B)^{-1},$$

where ω_V is the canonical sheaf of V.

Remark 0.1. A relatively minimal fibration is a fibration such that ω_f is f-nef. A semistable fibration is a relatively minimal fibration whose fibers are nodal and reduced.

The basic invariants for a relatively minimal fibration $f: X \to B$ are

$$(\omega_f \cdot \omega_f), \quad \deg f_*\omega_f \quad \text{and} \quad e_f := e(X) - e(B)e(F),$$

where e is the topological Euler number. These invariants are related by Noether's formula,

$$(\omega_f \cdot \omega_f) = 12 \deg f_*\omega_f - e_f.$$

It is well known that all these invariants are greater than or equal to 0; moreover, $\deg f_*\omega_f = 0$ if and only if f is locally trivial, $(\omega_f \cdot \omega_f) = 0$ implies that f is isotrivial, and $e_f = 0$ if and only if the fibration is smooth.

Received August 2, 2007. Revision received September 20, 2007.

Research partially supported by: PRIN 2003 *Spazi di moduli e teoria di Lie*; GNSAGA; FAR 2002 (Pavia) *Varietà algebriche, calcolo algebrico, grafi orientati e topologici.*