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# Asymptotic Expansion of the Heat Kernel for Orbifolds

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## 1. Introduction

Roughly speaking, a topological *orbifold* is a space locally homeomorphic to an orbit space of a finite group action on $\mathbb{R}^n$. A smooth orbifold consists of a Hausdorff second countable topological space together with an atlas of coordinate charts realizing such local homeomorphisms and satisfying compatibility conditions (see Section 2). Orbifolds were introduced by Satake and then studied by Thurston because of their utility in the investigation of 3-manifolds (e.g., a Seifert fibred 3-manifold is naturally a generalized circle bundle over a 2-orbifold); today, orbifolds arise naturally in diverse branches of mathematics and physics, including symplectic geometry, string theory, and vertex operator algebras.

We will be interested in orbifolds from a spectral-theoretic point of view. An orbifold endowed with a metric structure is a *Riemannian orbifold*. As in the manifold case, associated with every Riemannian metric is a Laplace operator acting on smooth functions on the orbifold. In the case of closed orbifolds, the Laplacian has a discrete spectrum. We study the relationship between the geometry and the Laplace spectrum of a closed orbifold via its heat kernel; as in the manifold case, the time-zero asymptotic expansion of the heat kernel furnishes geometric information about the orbifold.

Orbifolds began appearing sporadically in the spectral theory literature in the early 1990s and have received more concentrated attention in the last five years. Farsi [15] showed that the spectrum of an orbifold determines its volume by proving that Weyl's asymptotic formula holds for orbifolds. Dryden and Strohmaier [13] showed that, for a compact and negatively curved two-dimensional orbifold, the Laplace spectrum determines both the length spectrum and the orders of the singular points and vice versa; on the other hand, Doyle and Rossetti [12] gave (disconnected) examples of isospectral flat two-dimensional orbifolds with different length spectra and orders of singular points. Further investigations of the relationship between the lengths of closed geodesics and the spectrum were carried out by Stanhope and Uribe in [29]. It is natural to ask about the singularities that can appear in an isospectral family of orbifolds. Stanhope [28] showed that,

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