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An Intrinsic Characterization of the Unit Polydisc

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1. Introduction

Let M be a connected complex manifold and $\mathrm{Aut}(M)$ the group of all biholomorphic automorphisms of M. Then, equipped with the compact-open topology, $\mathrm{Aut}(M)$ is a topological group acting continuously on M.

In 1907 it was shown by Poincaré [15] that the Riemann mapping theorem does not hold in the higher-dimensional case. In fact, he proved that *there exists no biholomorphic mapping from the unit polydisc* Δ^2 *onto the unit ball* B^2 *in* $\mathbb{C}^2$ by comparing carefully the topological structures of the isotropy subgroups of $\mathrm{Aut}(\Delta^2)$ and $\mathrm{Aut}(B^2)$ at the origin o of $\mathbb{C}^2$. In view of this fact, for a given complex manifold M it is an interesting problem to bring out some complex analytic nature of M under some topological conditions on $\mathrm{Aut}(M)$.

In connection with this problem, in this paper we would like to study the following question.

Question. Let M and N be connected complex manifolds and assume that their holomorphic automorphism groups $\mathrm{Aut}(M)$ and $\mathrm{Aut}(N)$ are isomorphic as topological groups. Then, is M biholomorphically equivalent to N?

Recall that there exist relatively compact strictly pseudoconvex domains D_t $(t \in \mathbb{R})$ in a complex manifold X such that D_s is not biholomorphically equivalent to D_t unless $s = t$, and further, the only holomorphic automorphism of D_t is the identity for every t (see [3]). Thus, the answer to our question is negative, in general. However, there already exist several articles solving this question affirmatively in the case where the manifolds M or N are some special domains in $\mathbb{C}^n$ (see e.g. [4; 5; 6; 10; 11]). In particular, as an application of the classification theorem obtained by Isaev and Kruzhilin [6] for complex manifolds of dimension n admitting effective actions of the unitary group $U(n)$, Isaev [5] showed that *if the holomorphic automorphism group* $\mathrm{Aut}(M)$ *of a connected complex manifold* M *of dimension* n *is isomorphic to the holomorphic automorhism group* $\mathrm{Aut}(B^n)$ *of the unit ball* B^n *in* $\mathbb{C}^n$ *as topological groups, then* M *is biholomorphically equivalent to* B^n. In view of this, it would naturally be expected that exactly the same conclusion is

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