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The Space of Doubly Periodic Minimal Tori with Parallel Ends: Standard Examples

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1. Introduction

Scherk [10] presented in 1835 the first properly embedded minimal surface in $\mathbb{R}^3$ that was invariant by two linearly independent translations; we will shorten by saying a *doubly periodic minimal surface*. (Unless explicitly mentioned, all surfaces in this paper are presumed to be connected and orientable.) This surface is known as *Scherk's first surface,* and it fits naturally into a 1-parameter family $\mathcal{F} = \{F_\theta\}_\theta$ of examples known as doubly periodic Scherk minimal surfaces. In the quotient by its more refined period lattice (i.e., the period lattice generated by its shortest period vectors), each F_θ has genus 0 and four asymptotically flat annular ends: two top and two bottom ones, provided that the period lattice is horizontal. This kind of annular end is called a *Scherk-type end.* The parameter θ in this family $\mathcal{F}$ is the angle between top and bottom ends of F_θ. We can clearly consider the quotient of these F_θ by a less refined period lattice to have two top and $2k$ bottom ends for any natural k, keeping genus 0 in the quotient. Lazard-Holly and Meeks [5] proved that these are the only possible examples in this setting; that is, if the quotient of a doubly periodic minimal surface $M \subset \mathbb{R}^3$ has genus 0, then M must be a doubly periodic Scherk minimal surface up to translations, rotations, and homotheties. Moreover, the angle map $\theta : \mathcal{F} \to (0,\pi)$ is a diffeomorphism. Hence the moduli space of properly embedded minimal surfaces with genus 0 in $\mathbb{T} \times \mathbb{R}$, $\mathbb{T}$ a flat torus, is diffeomorphic to $(0,\pi)$ after identifying by rotations, translations, and homotheties.

In 1988, Karcher [3] defined another 1-parameter family of doubly periodic minimal surfaces, called *toroidal half-plane layers,* with genus 1 and four Scherk-type parallel ends in its smallest fundamental domain (these examples will be denoted as $M_{\theta,0,0}$ in Section 2). Furthermore, he exposed two distinct 1-parameter deformations of each toroidal half-plane layer and so obtained other doubly periodic minimal tori with parallel ends (denoted as $M_{\theta,\alpha,0}$ and $M_{\theta,0,\beta}$, with $\beta < \theta$, in Section 2). We generalize these Karcher's examples in Section 2 by obtaining a 3-parameter family.

Theorem 1. *There exists a* 3*-parameter family* $\mathcal{K} = \{M_{\theta,\alpha,\beta}\}_{\theta,\alpha,\beta}$ *of properly embedded doubly periodic minimal surfaces with genus* 1 *and four parallel ends*

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