Michigan Math. J. 54 (2006)

Hölder Estimates on Lineally Convex Domains of Finite Type

KLAS DIEDERICH & BERT FISCHER

1. Introduction

In [DFo1] Diederich and Fornæss constructed support functions for convex domains of finite type. This result together with a good knowledge of the local geometry of convex domains of finite type has been used in [DFFo] to prove optimal Hölder estimates for a solution of the Cauchy–Riemann equation in such domains. Hefer [H] has extended and refined this result by using information on the multitype of the domain. Further results on solution operators based on the support functions of [DFo1] can be found in [A; DM1; DM2; F1].

A different approach is attributable to Cumenge: in [Cu1] and [Cu2] she uses the Bergman kernel to construct solution operators for the Cauchy–Riemann equation in convex domains of finite type. Using these operators and some precise estimates (due to McNeal [Mc]) of the Bergman kernel at the boundaries of the domains, she also proves optimal Hölder estimates and some other results.

All results mentioned so far have one feature in common. The hypotheses on the given functions and forms and the conclusions of the papers are isotropic, whereas the proofs are quite nonisotropic in that the respective estimates necessarily take into account the different behavior of the geometry of the domains in different directions, even of the holomorphic tangent spaces. Introducing nonisotropy into the conclusions was first considered in [F2], where optimal nonisotropic Hölder estimates for certain solutions of the $\bar{\partial}$-equation are proved for bounded data.

Recently the larger class of lineally convex domains of finite type has received much attention. A smooth lineally convex domain differs from a (linearly) convex domain in that, with the former, only the complex tangent space at each boundary point is supposed to lie outside of the domain.

Diederich and Fornæss [DFo2] have constructed a smooth family of holomorphic support functions with best possible nonisotropic estimates for lineally convex domains of finite type. It has been shown by Conrad [C] that the local geometry of such domains shares all essential properties with convex domains of finite type.

In this paper we shall use these support functions to establish both isotropic and nonisotropic Hölder estimates on lineally convex domains of finite type. In doing so we can follow rather closely the proofs given in [DFFo; F2] for the corresponding results in the linearly convex case.

Received February 7, 2005. Revision received July 19, 2005.