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The Action of Geometric Automorphisms of Asymptotic Teichmüller Spaces

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1. Introduction

The Teichmüller space $T(R)$ of a Riemann surface R is a deformation space of the complex structure of R, and the quasiconformal mapping class group $\mathrm{MCG}(R)$ of R is the set of all homotopy classes of quasiconformal automorphisms of R. The quasiconformal mapping class group $\mathrm{MCG}(R)$ acts on the Teichmüller space $T(R)$ isometrically with respect to the Teichmüller distance, which induces the Teichmüller modular group $\mathrm{Mod}(R)$. If R is of analytically infinite type, then $T(R)$ is infinite dimensional and the action of $\mathrm{MCG}(R)$ on $T(R)$ is, in general, not discontinuous. This is equivalent to the orbit of some point in $T(R)$ under the action of $\mathrm{MCG}(R)$ not being discrete. This phenomenon appears only when Teichmüller spaces are infinite dimensional; that is, it does not occur for Riemann surfaces of analytically finite type. On the basis of this fact, in [8] and [9] we introduced the notion of limit sets and regions of discontinuity of Teichmüller modular groups (analogous to the theory of Kleinian groups) and studied the dynamics of Teichmüller modular groups. This paper applies the theory of dynamics to isometric automorphisms on a certain quotient space of $T(R)$ that we call the asymptotic Teichmüller space.

The asymptotic Teichmüller space $AT(R)$ of R was introduced in [14] for R the upper half-plane and in [4] and [13] for R an arbitrary hyperbolic Riemann surface. The asymptotic Teichmüller space $AT(R)$ is of interest only when R is of analytically infinite type; otherwise, $AT(R)$ consists of just one point. Similarly to the action of $\mathrm{MCG}(R)$ on $T(R)$, every element of $\mathrm{MCG}(R)$ induces an isometric automorphism of $AT(R)$. In particular, we have a homomorphism $\iota_{AT} : \mathrm{MCG}(R) \to \mathrm{Isom}(AT(R))$. We define the geometric automorphism group $\mathcal{G}(R)$ as the image $\iota_{AT}(\mathrm{MCG}(R))$.

We investigate the dynamical behavior of $\mathcal{G}(R)$ on $AT(R)$. However, it is different from the action of $\mathrm{MCG}(R)$ on $T(R)$ that the homomorphism ι_{AT} is not injective. Furthermore, the action of $\mathrm{MCG}(R)$ on $AT(R)$ can be trivial. In fact, there exists an example where the action of $\mathrm{MCG}(R)$ is trivial even though $AT(R)$ is nontrivial. It is therefore necessary to know when the action of $\mathrm{MCG}(R)$ on $AT(R)$ is nontrivial. We prove that if a Riemann surface R is of topologically infinite type and satisfies the upper bound condition, then the action of $\mathrm{MCG}(R)$ on

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