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# Domains in Almost Complex Manifolds with an Automorphism Orbit Accumulating at a Strongly Pseudoconvex Boundary Point

Kang-Hyurk Lee

## 1. Introduction

Let $(M, J)$ be an almost complex manifold and let $\Omega$ be a domain in $M$. Call $p \in \partial\Omega$ a *strongly J-pseudoconvex boundary point* if there is a $C^2$ local defining function whose Levi form is positive definite for the *J-complex tangent vector space* $T_p^J \partial\Omega = T_p\partial\Omega \cap JT_p\partial\Omega$ of $\partial\Omega$ at $p$. For $p \in \Omega$ and a sequence $\varphi^\nu \in \operatorname{Aut}(\Omega, J)$, call the sequence $\{\varphi^\nu(p) : \nu = 1, 2, \dots\}$ an *automorphism orbit* of $\Omega$. This paper pertains to the following problem.

> *Classify the domains $\Omega$ in an almost complex manifold $(M, J)$ that admit an automorphism orbit accumulating at a strongly J-pseudoconvex boundary point.*

In the complex case, the Wong–Rosay theorem states that such domains are biholomorphically equivalent to the unit ball $\mathbb{B}_n$ in $\mathbb{C}^n$ (see [3; 5; 10; 19; 22]). For the real 4-dimensional almost complex case, Gaussier and Sukhov [7] have shown that under a certain restriction such $(\Omega, J)$ is biholomorphic to the unit ball $\mathbb{B}_2$ in $\mathbb{C}^2$. But when $\dim M \geq 6$ it turns out that there are infinitely many biholomorphically distinct domains, as the following example shows.

Example 1.1. Let $z_j = x_j + iy_j$ be the standard coordinate functions of $\mathbb{C}^3 \simeq \mathbb{R}^6$. Set $z' = (z_2, z_3)$ and $z = (z_1, z')$. Let $\rho_t(z) = \operatorname{Re} z_1 + t|z'|^2$ and let

$$J_t(x) = \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & tx_2 \\ 1 & 0 & 0 & 0 & tx_2 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

for $t \in \mathbb{R}$. Consider the domain $\mathbb{H}_t = \{z \in \mathbb{C}^3 : \rho_t(z) < 0\}$ equipped with the almost complex structure $J_1$. It turns out that $(\mathbb{H}_t, J_1)$ with $t > 1/8$ has automorphisms $\Lambda_k(z) = \left(z_1/k, z'/\sqrt{k}\right)$, which induces an orbit accumulating at 0 that is

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