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Proper Holomorphic Maps between Reinhardt Domains in $\mathbb{C}^2$

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0. Introduction

Let D_1, D_2 be bounded Reinhardt domains in $\mathbb{C}^2$ and let $f : D_1 \to D_2$ be a proper holomorphic map. Such maps are often *elementary algebraic*; that is, have the "monomial" form

$$z \mapsto \text{const}\, z^a w^b,$$

$$w \mapsto \text{const}\, z^c w^d,$$

where z, w denote variables in $\mathbb{C}^2$ and where a, b, c, d are integers such that $ad - bc \neq 0$. For brevity we shall call such maps *elementary* maps. All elementary maps are well-defined outside I, the union of the coordinate complex lines, but not necessarily at points in I. The question of the existence of an elementary proper holomorphic map between two given domains is resolved by passing to the logarithmic diagrams of the domains. Several classes of domains between which only elementary proper holomorphic maps are possible have been described in [S].

The aim of this paper is to identify situations in which f is not elementary and to explicitly describe all forms that the map f and the domains D_1, D_2 may have in such cases. If f is biholomorphic, then it can be represented as the composition of an elementary biholomorphism between D_1 and D_2 and automorphisms of these domains (see [Kr; Sh]). Therefore, nonelementary biholomorphisms can occur only between domains that are equivalent by means of an elementary map and having nonelementary automorphisms (and that are hence straightforward to determine).

Proper maps that are not biholomorphic are harder to deal with. Nonelementary maps may occur, for example, if each of D_1, D_2 is a bidisc, in which case at least one component of f contains a Blaschke product with a zero away from the origin. In [BeP] and [LS], the problem of describing nonelementary proper holomorphic maps was studied for complete Reinhardt domains; it turns out that, apart from the example of bidiscs, such maps can arise only if D_1 and D_2 are certain pseudo-ellipsoids. On the other hand, all proper holomorphic maps between pseudo-ellipsoids in $\mathbb{C}^n$ for $n \geq 2$ can be found using arguments from [D-SP]. All

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