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The Rank-2 Lattice-Type Vertex Operator Algebras V_L^+ and Their Automorphism Groups

Chongying Dong & Robert L. Griess, Jr.

1. Introduction

This article continues a program to study automorphism groups of vertex operator algebras (VOAs). See references in the survey [G2] and the more recent articles [G3], [DG1], [DG2], [DGR], and [DN1].

Here we investigate the fixed point subVOA of a lattice-type VOA with respect to a group of order 2 lifting the -1 map on a positive definite lattice. We can obtain a definitive answer for the automorphism group of this subVOA in two extreme cases. The first is where the lattice has no vectors of norm 2 or 4, and the second is where the lattice has rank 2.

We use the standard notation V_L for a lattice VOA based on the positive definite even integral lattice L. For a subgroup G of $\mathrm{Aut}(L)$, V_L^G denotes the subVOA of points fixed by G. When G is a group of order 2 lifting -1_L, it is customary to write V_L^+ for the fixed points (even though, strictly speaking, G is defined only up to conjugacy; see the discussion in [DGH] or [GH]).

The rank-2 case is a natural extension of work on the rank-1 case, where $\mathrm{Aut}(V_L^G)$ was determined for all rank-1 lattices L and all choices of finite group $G \leq \mathrm{Aut}(V_L)$. The styles of proofs are different. In the rank-1 case, there was heavy analysis of the representation theory of the principal Virasoro subVOA on the ambient VOA. In the rank-2 case, there is a lot of work on idempotents and solving nonlinear equations as well as work with several subVOAs associated to Virasoro elements. For rank 2, the case of nontrivial degree-1 part is harder to settle than in rank 1.

Our strategy follows this model. Let V be one of our V_L^+. We get information about $G := \mathrm{Aut}(V)$ by its action on the finite-dimensional algebra $A := (V_2, 1^{st})$. We take a subset S of A that is G-invariant and understand S well enough to limit the possibilities for G (usually, there are no automorphisms besides the ones naturally inherited from V_L). A natural choice for S is the set of idempotents or conformal vectors. Usually, S spans A or at least generates A. In the main case of a rank-2 lattice, we prove that $\mathrm{Aut}(V)$ fixes a subalgebra of A that is the natural $M(1)_2^+$. The structure of V is controlled by $M(1)^+$, which is generated by $M(1)_2^+$ and its eigenspaces, so we eventually determine G.

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