Michigan Math. J. 53 (2005)

Detonation Waves in a Transverse Magnetic Field

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1. Preliminaries

The equations describing the evolution of a combustible gas display a rich variety of nonlinear phenomena, including those encountered in reaction diffusion equations and in shock wave theory. In particular, there are two distinct mechanisms that can lead to the formation of combustion waves. *Deflagration* occurs when an exothermic chemical reaction is initiated in a heat-conducting gas. The subsequent diffusion of liberated heat into the surrounding medium leads to the formation of a flame that propagates into the unburned region. The fluid dynamics of the gas mixture play a negligible role in this regime. Fast combustion, or *detonation,* occurs in a dramatically different manner. Here, the process is initiated by a strong fluid dynamical shock layer that propagates into the unburned region. If the shock is sufficiently strong then the gas will be heated above its ignition temperature, causing the gas to burn in a reaction zone behind the shock. In other words, detonation waves are compressive, exothermically reacting shock waves whereas deflagration waves are expansive shock waves. For the exothermic, irreversible reactions considered here, strong deflagrations violate the second law of thermodynamics and are unphysical, and weak detonations are rare. If we permit an endothermic region then strong deflagrations and weak detonations are possible and perhaps even probable [7]. So deflagration waves will not be discussed in this paper. When the shock wave amplitude is minimum, the detonation wave is in the Chapman–Jouguet regime and plays a special role in magnetofluiddynamics, as in ordinary fluid dynamics or classical aerodynamics. This regime is realized when the shock wave occurs as a result of the energy released in a chemical reaction. The effect of this released energy appears in the conservation law of energy, which is in the related system of conservation law equations (see [2; 5; 6]).

Now we consider a one-step exothermic chemical reaction as Reactant $\to$ Product occurring in the presence of magnetic and electrical fields. The equations governing the reaction flow are

$$(\lambda_1 + 2\mu_1)\frac{du}{dx} = mu + p + \frac{1}{2}\mu(H_y^2 + H_z^2) - P,$$

$$\mu_1\frac{dv}{dx} = mv - \mu H_x H_y - P_1,$$

Received July 29, 2004. Revision received December 8, 2004.