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On Cohomology of Invariant Submanifolds of Hamiltonian Actions

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1. Introduction

In [5] the author proved that if there is a free algebraic circle action on a nonsingular real algebraic variety X then the fundamental class is trivial in any nonsingular projective complexification $i: X \to X_{\mathbb{C}}$. The Kähler forms on $\mathbb{C}^N$ and $\mathbb{C}P^N$ naturally induce symplectic structures on complex algebraic affine or projective varieties and, when defined over reals, their real parts (if not empty) are Lagrangian submanifolds.

The following result can be considered as a symplectic equivalent, on real algebraic varieties, of the result just described.

Theorem 1.1. *Assume that G is S^1 or S^3 acting on a compact symplectic manifold (M, ω) in a Hamiltonian fashion, and assume that L^l is an invariant closed submanifold. If the G-action on L is locally free then the homomorphism induced by the inclusion $i: L \to M$,*

$$H_i(L, \mathbb{Q}) \to H_i(M, \mathbb{Q}),$$

is trivial for $i \geq l - k + 1$, where $k = \dim(G)$. In particular, the fundamental class $[L]$ is trivial in $H_l(M, \mathbb{Q})$.

Moreover, if the corresponding sphere bundle $S^k \to L \times EG \to L_G$ has nontorsion Euler class then the homomorphism

$$i_*: H_{l-k}(L, \mathbb{Q}) \to H_{l-k}(M, \mathbb{Q}),$$

induced by the inclusion $i: L \to M$, is also trivial (see Section 2 for the definition of EG and L_G).

Since any compact connected Lie group has a circle subgroup, we deduce the following immediate corollary.

Corollary 1.2. *Let G be a compact connected Lie group acting on a compact symplectic manifold (M, ω) in a Hamiltonian fashion, and let L be an invariant closed submanifold of dimension l. If the G-action on L is locally free, then the fundamental class $[L]$ is trivial in $H_l(M, \mathbb{Q})$.*

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