Michigan Math. J. 53 (2005)

Kähler Submanifolds with Ricci Curvature Bounded from Below

Young Jin Suh & Hae Young Yang

1. Introduction

A complex n-dimensional ($n \geq 2$) Kähler manifold of constant holomorpic sectional curvature $c > 0$ is called a *complex projective space* and is denoted by $\mathbb{C}P^{n+p}(c)$. In this paper we want to study some complete Kähler submanifolds in a complex projective space $\mathbb{C}P^{n+p}(1)$ concerned with the Ricci curvatures.

The theory of Kähler submanifolds was systematically studied by Ogiue [5; 6; 7]; in [6], some pinching problems concerned with the Ricci curvatures were studied. Specifically, the following theorem was proved.

Theorem A. *Let M be an n-dimensional complete Kähler submanifold of an $(n+p)$-dimensional complex projective space $\mathbb{C}P^{n+p}(1)$ of constant holomorphic sectional curvature* 1. *If the Ricci curvatures are greater than $n/2$, then M is totally geodesic.*

Now let us consider a generalization of this theorem to the case where the Ricci curvature is greater than *or equal* to $n/2$. Before giving such a classification problem concerned with the Ricci curvature, we introduce the following theorem (due to Nakagawa and Takagi [4]) related to the parallel second fundamental form.

Theorem B. *Let M^n be a compact Kähler submanifold immersed in a complex projective space $\mathbb{C}P^m(1)$ with parallel second fundamental form. Then M is an imbedded submanifold congruent to the standard imbedding of one of the following submanifolds.*

Submanifold	*Dim*	*Codim*	*Scalar*
$M_1 = \mathbb{C}P^n(1)$	n	0	$n(n+1)$
$M_2 = \mathbb{C}P^n\left(\frac{1}{2}\right)$	n	$\frac{1}{2}n(n+1)$	$\frac{1}{2}n(n+1)$
$M_3 = \mathbb{C}P^{n-s}(1) \times \mathbb{C}P^s(1)$	n	$s(n-s)$	$s^2+(n-s)^2+n$
$M_4 = Q^n, n \geq 3$	n	1	n^2
$M_5 = U(s+2)/U(2) \times U(s), s \geq 3$	n	$\frac{1}{2}s(s-1)$	$2s(s+2)$
$M_6 = \mathrm{SO}(10)/U(5)$	10	5	80
$M_7 = E_6/\mathrm{Spin}(10) \times T$	16	10	192

Received June 7, 2004. Revision received August 17, 2005.
This work was supported by the grant Proj. No. R14-2002-003-01001-0 from Korea Research Foundation, Korea.