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C^k-Estimates for the $\bar\partial_b$-Equation on Convex Domains of Finite Type

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1. Introduction

Since the construction in [8] of a support function for convex domains of finite type, many results about the regularity of Cauchy–Riemann equations have been obtained on these domains. We should mention [7], in which a $\bar\partial$-solving operator for all convex domains of finite type was constructed that satisfies optimal uniform Hölder estimates. Note that this result was already obtained in [5] by using properties of the Bergman kernel. For a convex domain of finite type, Hefer [12] obtained Hölder and L^p-estimates depending on Catlin's multitype. In [2], a modification of the operator of [7] led to C^k-estimates for all $k \in \mathbb{N}$. In this work, we are interested in the regularity of tangential Cauchy–Riemann equations.

Let D be a bounded convex domain in $\mathbb{C}^n$ of finite type m, with bD its boundary. We denote by r a C^∞-defining convex function for D such that $\operatorname{grad} r(\zeta) \neq 0$ for all ζ in a neighborhood $\mathcal{V}$ of bD. We use the definition of the equivalence classes and of the $\bar\partial_b$ operator given in [13] and denote by $[f]$ the class of a form f.

Let $C^\alpha_{0,q}(bD)$, $\alpha \geq 0$, be the set of $(0,q)$-forms of regularity C^α in a neighborhood of bD and let $\tilde C^\alpha_{0,q}(bD)$ be the set of equivalence classes $[f]$ such that $f \in C^\alpha_{0,q}(bD)$. The tangential norm $\|[f]\|_{bD,\alpha}$ is then defined by

$$\|[f]\|_{bD,\alpha} := \inf\{\|g\|_{bD,\alpha},\ g \in C^\alpha_{0,q}(bD),\ [g] = [f]\}.$$

Now we state our main result.

Theorem 1.1. *Let D be a bounded convex domain with C^∞-smooth boundary of finite type m in $\mathbb{C}^n$, and let $q = 1, \dots, n-1$. Then there exist two linear operators $[T_q], [\tilde T_q] : \tilde C^0_{0,q}(bD) \to \tilde C^0_{0,q-1}(bD)$ such that the following statements hold.*

(i) *For all $k \in \mathbb{N}$ there is a constant $c_k > 0$ such that, for all $[f] \in \tilde C^k_{0,q}(bD)$, $[T_q][f]$ and $[\tilde T_q][f]$ are in $\tilde C^{k+1/m}_{0,q-1}(bD)$ and*

$$\|[\tilde T_q][f]\|_{bD,k+1/m} + \|[T_q][f]\|_{bD,k+1/m} \leq c_k \|[f]\|_{bD,k}.$$

(ii) *For all $[f] \in \tilde C_{0,q}(bD)$ such that $\bar\partial_b[f]$ belongs to $\tilde C_{0,q+1}(bD)$ and with the additional hypothesis when $q = n-1$ that $\int_{bD} f \wedge \phi = 0$ for all $\bar\partial$-closed forms $\phi \in C^\infty_{n,0}(bD)$, we have*

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