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On the Pfister–Leep Conjecture on C_0^d-Fields

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1. Introduction

In analogy to algebraically closed fields, a field k is called a *C_0^d-field* if every system of r homogeneous forms of degree d over k in n variables ($n > r$) has a common nontrivial zero over k. For a prime p, a field k is called a *p-field* if $[L : k]$ is a power of p for every finite extension L/k.

In [3], Pfister proves the following theorem.

Theorem [3, Thm. 2]. *If k is a p-field then, for any d not divisible by p, k is a C_0^d-field.*

See also [4, Thm. 2]. A special case is as follows.

Corollary [3, Cor. 1]. *If k is a p-field for some prime $p \neq 2$, then k is a C_0^2-field.*

Pfister conjectured that the converse of this corollary is true.

Pfister's Conjecture [3, Conjecture 3]. *If k is a C_0^2-field, then k is a p-field for some prime $p \neq 2$.*

In [2, Thms. 5.4 & 5.5], Leep proved this conjecture for fields of characteristic 0 or 2 and gave the following generalized version of Pfister's conjecture to higher-degree forms (see [2, 1.4]).

The Conjecture of Pfister–Leep. *For a fixed d, if k is a C_0^d-field then k is a p-field for some prime $p \nmid d$.*

In this note we show (Corollary 3.2) that the Pfister–Leep conjecture is true if d is a power of the characteristic of the field k. Note that if k is a $C_0^{q^i}$-field then k is also a C_0^q-field (because if $\{F_1, \dots, F_r\}$ is a system of forms of degree q, then $\{F_1^{q^{i-1}}, \dots, F_r^{q^{i-1}}\}$ is an equivalent system of forms of degree q^i). Therefore, we need only consider the case when d is equal to the characteristic of k.

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