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Edited 4Θ-Embeddings of Jacobians

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1. Introduction

The point of departure for this paper is the elementary algebraic construction of Jacobians given in [A]. We begin by reviewing that construction. For brevity's sake we reformulate the construction in terms of line bundles rather than divisors. Let X be a nonsingular projective algebraic curve of genus $g > 0$. Although the main work of this paper takes place over the complex numbers, for the moment we take as ground field any algebraically closed field. Fix an integer $n \geq g + 2$. For $i = 0, \dots, n+1$, let $f \mapsto f^{(i)}$ denote the operation of pull-back via the ith projection $X^{\{0,\dots,n+1\}} \to X$. Fix a line bundle $\mathcal{E}$ on X of degree $n + g - 1$. Given any line bundle $\mathcal{T}$ on X of degree 0, let u (resp. v) be a row vector of length n with entries forming a basis of $H^0(X, \mathcal{T}^{-1} \otimes \mathcal{E})$ (resp. $H^0(X, \mathcal{T} \otimes \mathcal{E})$) over the ground field, and let $\operatorname{abel}(\mathcal{T})$ be the $n \times n$ matrix with entries

$$\operatorname{abel}(\mathcal{T})_{ij} := \begin{vmatrix} \widehat{v^{(0)}} \\ \vdots \\ \widehat{v^{(i)}} \\ \vdots \end{vmatrix} \cdot \begin{vmatrix} \vdots \\ \widehat{u^{(i)}} \\ \vdots \\ \widehat{u^{(n+1)}} \end{vmatrix} \cdot \begin{vmatrix} \vdots \\ \widehat{v^{(j)}} \\ \vdots \\ \widehat{v^{(n+1)}} \end{vmatrix} \cdot \begin{vmatrix} \widehat{u^{(0)}} \\ \vdots \\ \widehat{u^{(j)}} \\ \vdots \end{vmatrix},$$

where the leftmost determinant is that obtained by (i) stacking the row vectors $v^{(i)}$ to form an $(n+2) \times n$ matrix with rows numbered from 0 to $n+1$, then (ii) striking row 0 and row i to obtain a square matrix, and (iii) finally taking the determinant; the other determinants are analogously formed. Up to a nonzero scalar multiple, the matrix $\operatorname{abel}(\mathcal{T})$ is independent of the choice of bases u and v and moreover depends only on the isomorphism class of the line bundle $\mathcal{T}$. It is easy to see that $\operatorname{abel}(\mathcal{T})$ does not vanish identically. The construction $\mathcal{T} \mapsto \operatorname{abel}(\mathcal{T})$ maps the set of isomorphism classes of degree-0 line bundles on X to the projective space of lines in the space of $n \times n$ matrices with entry in ith row and jth column drawn from the space

$$H^0\left(X^{\{0,\dots,n+1\}}, \frac{\bigotimes_{\ell=0}^{n+1} (\mathcal{E}^{(\ell)})^{\otimes 4}}{(\mathcal{E}^{(0)})^{\otimes 2} \otimes (\mathcal{E}^{(i)})^{\otimes 2} \otimes (\mathcal{E}^{(j)})^{\otimes 2} \otimes (\mathcal{E}^{(n+1)})^{\otimes 2}}\right).$$

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