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Nonisotropic Hölder Estimates on Convex Domains of Finite Type

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1. Introduction

In [DFo], Diederich and Fornæss constructed a smooth support function for convex domains of finite type that satisfies the correct estimates to be used in the construction of several integral kernels. (See e.g. [DFFo; DM1; DM2; F; H].) In [DFFo] this support function was used to construct solution operators for the Cauchy–Riemann equation that satisfy optimal (isotropic) Hölder estimates, as follows.

Theorem 1.1 [DFFo]. *Let $D \subset\subset \mathbb{C}^n$ be a linearly convex domain with $\mathcal{C}^\infty$-smooth boundary of finite type m. We denote by $C^0_{(0,q)}(\bar{D})$ the Banach space of $(0,q)$-forms with continuous coefficients on $\bar{D}$ and by $\Lambda^{1/m}_{(0,q)}(D)$ the Banach space of $(0,q)$-forms whose coefficients are uniformly Hölder continuous of order $1/m$ on D. Then there are bounded linear operators*

$$T_q : C^0_{(0,q+1)}(\bar{D}) \to \Lambda^{1/m}_{(0,q)}(D)$$

such that $\bar{\partial} T_q f = f$ for all $f \in C^0_{(0,q+1)}(\bar{D})$ with $\bar{\partial} f = 0$.

In fact, a simple modification of the standard example shows that in general the solution of a Cauchy–Riemann equation with bounded left-hand side cannot be better than $(1/m)$-Hölder continuous. However, a closer look at the example shows that it is in the *normal* direction that the Hölder exponent cannot be better than $1/m$. On the other hand, Krantz [K] has shown that $(1/2)$-Hölder continuous solutions of the Cauchy–Riemann equation in strongly pseudoconvex domains are almost 1-Hölder continuous in the complex tangent directions, and a similar result is known for finite-type domains in $\mathbb{C}^2$ (see [ChK]). In our case the situation is even more difficult because we have several complex tangential directions and, in contrast to the strongly pseudoconvex case, these directions cannot be handled in an isotropic way. We expect nevertheless to derive a solution that is $(1/m)$-Hölder continuous in the normal direction and satisfies better estimates in the complex tangential directions.

It turns out that such estimates are best expressed in terms of a certain pseudometric that is associated to the given domain D. So let $D = \{\rho < 0\} \subset \mathbb{C}^n$ be a bounded convex domain with $\mathcal{C}^\infty$-boundary of finite type m. We further assume

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