Michigan Math. J. 52 (2004)

Fatou Sets for Rational Maps of $\mathbb{P}^k$

Kazutoshi Maegawa

1. Introduction

In this paper we study the dynamics of a rational self-map of $\mathbb{P}^k$. We deal with the Fatou set and give a rough classification of the dynamics on the Fatou components for a rational map whose indeterminacy set is nonempty. Our result is just the first step toward a complete classification of Fatou components, but it shows us a new dynamical phenomenon.

For an algebraically stable (AS) rational self-map f of degree ≥ 2 of $\mathbb{P}^k$, the Fatou set is defined to be the set of all the Lyapunov stable points for f. The connected components of the Fatou set are called Fatou components. The indeterminacy set I (and, moreover, the extended indeterminacy set E) are automatically disjoint from the Fatou set. Let $\mathrm{Orb}(x)$ denote the forward orbit of $x \in \mathbb{P}^k$. The main purpose of this paper is to show the following theorem. Keep in mind that it is not a direct consequence of the definition of the Fatou set because $\operatorname{codim}_{\mathbb{C}} I \geq 2$.

Theorem 1.1. *Let f be an AS rational self-map of degree ≥ 2 of $\mathbb{P}^k$, and let U be a Fatou component for f. If there exists at least one point $x \in U$ such that $\overline{\mathrm{Orb}(x)} \cap I \neq \emptyset$, then $\overline{\mathrm{Orb}(y)} \cap I \neq \emptyset$ for any $y \in U$.*

From this theorem it follows that there are two types of Fatou components in terms of the relationship between their respective limit maps and I. We call a Fatou component that contains no point whose forward orbit accumulates at I a *regular* Fatou component. We will show that any regular Fatou component is Stein. Further, we will give a formula that relates the Green (1, 1) current to the union of all regular Fatou components. This is an improvement on the theorem of Fornæss and Sibony in [FS].

Acknowledgment. This study was done as part of the author's dissertation. He thanks his advisor, Professor T. Ueda, for useful conversations as well as Professor J. E. Fornæss for his hospitality while the author visited the University of Michigan during the Fred and Lois Gehring special year in complex analysis.

Received May 28, 2002. Revision received May 6, 2003.
Partially supported by JSPS research fellowships for young scientists.