

Regularity of Continuous CR Maps in Arbitrary Dimension

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1. Introduction

In this article we come back to one of those questions of complex analysis that are at the same time natural, important for applications, easy to formulate, and yet quite intriguing in the sense that they have given rise to much research without having been solved completely. More specifically, we mean the following problem.

GENERAL PROBLEM. Let $D, D' \subset \subset \mathbb{C}^n$ be domains and let $f: D \rightarrow D'$ be a proper holomorphic map. Suppose that ∂D and $\partial D'$ have a certain regularity property (e.g., C^k -smooth for some $k = 1, 2, 3, \dots, \infty, \omega$). Does the map f extend automatically to a map $\hat{f}: \bar{D} \rightarrow \bar{D}'$ with some regularity depending on the regularity of the boundaries $\partial D, \partial D'$? (For instance, for which k do C^k -smooth boundaries imply that \hat{f} is C^k ?)

For $n = 1$ there is a quite precise understanding of these questions, whereas good answers for $n > 1$ are known only under additional hypotheses. The answers to the general questions are unknown even for $k = \infty$ and $k = \omega$.

The question of boundary regularity of proper holomorphic mappings is not only natural but also important as a tool for other questions. Namely, if any proper holomorphic map $f: D \rightarrow D'$ automatically has a sufficiently high boundary regularity, then this will imply that the local biholomorphic invariants of a real hypersurface are part of the geometry of the domains bounded by them. This can, for instance, be very useful for studying the existence of certain proper holomorphic maps and many other problems.

Extensive research has been done in the area of the General Problem. We cannot mention it in full detail. Instead, we refer the reader to existing survey articles on the subject (e.g. [8; 22]).

In this article we deal, more specifically, with the case $k = \omega$ of the General Problem. We want to know whether this implies that \hat{f} necessarily is C^ω or, in other terms, whether all proper holomorphic maps $f: D \rightarrow D'$ extend holomorphically to an open neighborhood of \bar{D} if ∂D and $\partial D'$ are C^ω -smooth. We will show that the answer to this question is indeed “yes” if one knows already that f

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