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A Generalization to the q-Convex Case of a Theorem of Fornæss and Narasimhan

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1. Introduction

Fornæss and Narasimhan proved (in [8, Thm. 5.3.1]) that, for any complex space X, the identity $\mathrm{WPSH}(X) = \mathrm{PSH}(X)$ holds, where $\mathrm{WPSH}(X)$ denotes the weakly plurisubharmonic functions on X and $\mathrm{PSH}(X)$ denotes, as usual, the plurisubharmonic functions on X.

When X has no singularities, this identity is clear. For the singular case, however, the inclusion $\mathrm{WPSH}(X) \subseteq \mathrm{PSH}(X)$ is no longer trivial; one must find locally a plurisubharmonic extension to the ambient space of an embedding of X.

In this paper we give another proof for this identity (Theorem 3.3). It is shorter and easier and has the advantage that it can be generalized to q-plurisubharmonic functions (Theorem 4.16). However it has the disadvantage that it works only for continuous functions. The q-plurisubharmonic functions were introduced by Hunt and Murray in [10] (see also [9]), but we will call here q-plurisubharmonic what they call $(q-1)$-plurisubharmonic.

We also obtain a generalization of a theorem of Siu [16]; namely, we show (Lemma 4.18) that every q-complete subspace with corners of a complex space X admits a neighborhood in X that is q-complete with corners. This will be needed in the proof of our main result.

The results and proofs of this paper have been announced in [13]. This paper is part of the author's doctoral thesis written in Wuppertal. I thank Prof. M. Colţoiu and Prof. K. Diederich for many helpful discussions during the whole time of preparing my thesis. I thank the Department of Mathematics of the University of Wuppertal for providing me a nice working atmosphere.

2. Preliminaries

Let X be a complex space (with singularities). We denote by $\mathrm{PSH}(X)$ the plurisubharmonic functions on X. We use $\mathrm{SPSH}(X)$ to denote the *strongly plurisubharmonic functions* on X, that is, those PSH functions for which we have: for every $\theta \in \mathcal{C}_0^\infty(X, \mathbb{R})$, there exists an $\varepsilon_0 > 0$ such that $\varphi + \varepsilon\theta \in \mathrm{PSH}(X)$ for $0 \le \varepsilon \le \varepsilon_0$.

We will denote by $\mathrm{WPSH}(X)$ the class of *weakly plurisubharmonic functions* on X (as they are defined in [8]), that is, the class of upper semicontinuous functions $\varphi\colon X \to [-\infty, \infty)$ such that, for any holomorphic function $f\colon \Delta \to X$

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