# RELATIONS BETWEEN INTEGRAL AND MODULAR REPRESENTATIONS

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## 1. INTRODUCTION

Throughout this paper, let R denote a noetherian complete local integral domain, with maximal ideal P, residue class field $\bar{R} = R/P$, and field of quotients K. For example, one possible choice for R might be a valuation ring in some p-adic field. Let A be an R-algebra with unity element 1, finitely generated as R-module. An A-module will mean a left A-module, finitely generated over R, on which 1 acts as identity operator.

Set $\bar{A} = A/PA$, a finite-dimensional $\bar{R}$-algebra. To each A-module M there corresponds an $\bar{A}$-module $\bar{M} = M/PM$. As is well known, the mapping $M \to \bar{M}$ gives a one-to-one isomorphism-preserving correspondence between projective A-modules M and projective $\bar{A}$-modules $\bar{M}$. One of the main results of the present work is a generalization of this theorem for the special case in which A is the group ring RG of a finite group G. This permits us to establish some relationships between representation algebras of RG-modules and those of $\bar{R}$G-modules.

Section 2 is devoted to the necessary preliminaries concerning R-algebras. Most of the results given there are already known but not readily available in any single reference. We have therefore outlined a few of the proofs, for the convenience of the reader.

In Section 3, after some easy results on A-modules, we restrict ourselves to the case A = RG, and obtain the above-mentioned generalization. The paper concludes in Section 4 with various propositions concerning the behavior of modules under ground ring extension. One of these gives a necessary and sufficient condition that an A-module be absolutely indecomposable. Another asserts that if $\bar{R}$ is a finite field and M an indecomposable A-module, then for each suitably restricted ring S containing R, the $S \otimes A$-module $S \otimes M$ splits into a direct sum of indecomposable submodules, no two of which are isomorphic.

## 2. ALGEBRAS OVER COMPLETE LOCAL RINGS

In this section we list a number of results about algebras over complete local rings. We draw heavily from Jacobson [8]; but we simplify his proofs, because we do not need his results in the full generality with which he presents them. Other relevant references are Azumaya [1], Borevič and Faddeev [2], Conlon [3], Curtis and Reiner [4], Green [5], Swan [13].

If A is an arbitrary ring with 1, denote by rad A its Jacobson radical (see Jacobson [8, Chapter I]). Then rad A is a two-sided ideal of A, and the factor ring A/rad A has zero radical.

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