AN ALGEBRA RELATED TO THE ORTHOGONAL GROUP

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INTRODUCTION

I wish to express my sincere gratitude to Professor R. M. Thrall, under whose guidance the work presented here was done. I am also grateful to Professor R. Brauer, who suggested the problem, and to Professor H. Weyl. The extent to which I relied upon their earlier work will be apparent.

The question of commuting algebras of representations of classical groups over tensor space has been discussed by Brauer [2] and Weyl [5, Chapters III, IV and V]. In the case of the general linear group, the algebra concerned is the group algebra of the symmetric group. Much information concerning the representation theory of the general linear group is obtained from the corresponding theory of the symmetric group. Brauer [1] has defined a number of algebras which replace the symmetric group algebra when the general linear group is replaced by certain subgroups.

This paper is concerned with the algebra which arises in the case of the orthogonal group. Its definition by means of diagrams is taken from Brauer's paper [2]. In the first three chapters, results concerning the structure of the algebra are obtained directly. The fourth chapter makes contact with Weyl's results concerning the representation of the algebra in tensor space [5, Chapter V, Section B].

CHAPTER I

THE ALGEBRA ω_f^n

1.1. A REPRESENTATION OF $\mathfrak{S}_f$ BY DIAGRAMS

A permutation $\sigma \in \mathfrak{S}_f$, the symmetric group on f symbols, may be represented by a diagram consisting of two rows of f dots, the dots of each row being associated with the integers 1, 2, ⋯, f, from left to right, and dot i of the lower row being joined to dot σi of the upper row $(i = 1, 2, \cdots, f)$. Multiplication of diagrams to obtain $\tau\sigma$ is performed by placing a diagram for τ with its lower row of dots coincident with the upper row of dots of the diagram for σ. In this way a composite diagram (τ, σ) is obtained in which dot i of the lower row joins dot σi of the middle row, which in turn joins dot $\tau(\sigma i)$ of the upper row. If multiplication of permutations is performed from right to left, then the new diagram, obtained by deleting the middle row and joining dot i directly to dot $\tau(\sigma i)$, is the diagram for $\tau\sigma$. Suppose for example that $\tau = (1\,3) \in \mathfrak{S}_3$ and $\sigma = (1\,3\,2) \in \mathfrak{S}_3$;

Received December 20, 1954.

This paper represents a major part of the author's doctoral dissertation, accepted by the University of Michigan in June, 1952. The work was done under the sponsorship of the United States Office of Naval Research, Contract N8-ONR 71400.