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Graphs of Holomorphic Functions with Isolated Singularities Are Complete Pluripolar

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1. Introduction

In classical potential theory one encounters the notions of polar set and complete polar set. A set $E \subset \mathbb{R}^n$ is called *polar* if there exists a subharmonic u on a neighborhood of E such that $E \subset \{x : u(x) = -\infty\}$; E is called *complete polar* if one actually has $E = \{x : u(x) = -\infty\}$. (The function identically equal to $-\infty$ is not considered to be subharmonic.) It is well known that we may take u to be defined on all of $\mathbb{R}^n$ and also that E is complete polar if and only if E is polar and a G_δ (cf. [5]).

In pluripotential theory the situation is more complicated. A set E in a domain $D \subset \mathbb{C}^n$ is called *pluripolar in D* if there exists a plurisubharmonic function u on D such that $E \subset \{z : u(z) = -\infty\}$; E is called *complete pluripolar in D* if, for some plurisubharmonic function u on D, we have $E = \{z : u(z) = -\infty\}$. Although Josefson's theorem [4] asserts that E being pluripolar in D implies that E is pluripolar in $\mathbb{C}^n$, the corresponding assertion is false in the complete pluripolar setting. Also, a pluripolar G_δ need not be complete: the open unit disk Δ in the complex line $z_2 = 0$ in $\mathbb{C}^2$ is a G_δ but is not complete in $\mathbb{C}^2$. In fact, every plurisubharmonic function on $\mathbb{C}^2$ that equals $-\infty$ on Δ must equal $-\infty$ on the line $z_2 = 0$. Thus, it is reasonable to introduce the *pluripolar hull* of a pluripolar set $E \subset D$ as

$$E^*_D = \left\{z \in D : u\big|_E = -\infty \implies u(z) = -\infty \ \forall u \in \mathrm{PSH}(D)\right\},$$

where $\mathrm{PSH}(D)$ denotes the set of all plurisubharmonic functions on D. We also have use for the *negative pluripolar hull*,

$$E^-_D = \left\{z \in D : u\big|_E = -\infty \implies u(z) = -\infty \ \forall u \in \mathrm{PSH}(D),\ u \le 0\right\}.$$

If E is complete pluripolar in D then clearly E is a G_δ and $E^*_D = E$. A partial converse is Zeriahi's theorem [11].

Theorem 1. *Let E be a pluripolar subset of a pseudoconvex domain D in $\mathbb{C}^n$. If $E^*_D = E$ and E is a G_δ as well as an F_σ, then E is complete pluripolar in D.*

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