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Davis's Inequality for Orthogonal Martingales under Differential Subordination

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1. Introduction

Consider two $\mathbb{H}$-valued semimartingales X and Y, where $\mathbb{H}$ is a separable Hilbert space with norm $|\cdot|$ and inner product $\langle\cdot,\cdot\rangle$. We denote by $\mathcal{F} = \{\mathcal{F}_t\}_{t\geq 0}$ their common filtration, which is a family of right-continuous sub-σ-fields in a probability space $\{\Omega, \mathcal{A}, P\}$. We also assume that $\mathcal{F}_0$ contains all the sets of probability zero. We use the notation $[X,Y] = \{[X,Y]_t\}_{t\geq 0}$ to denote the quadratic covariation process between X and Y (see e.g. [DM]). Unless otherwise stated, we assume that all semimartingales have right-continuous paths with left limits (r.c.l.l.). For notational simplicity, we use $[X] = \{[X]_t\}_{t\geq 0}$ to denote $[X,X]$.

Since all the results in the paper are invariant under Hilbert space isomorphisms, we can restrict to the spaces of square integrable sequences.

We say that Y is *differentially subordinate* to X if $[X]_t - [Y]_t$ is nondecreasing and nonnegative as a function of t. A slightly weaker notion of martingale differential subordination was first introduced by Burkholder for discrete-time martingales and certain stochastic integrals (see [Bu1; Bu2; Bu3; Bu4; Bu5; Bu6] for connections and applications to various settings in Banach spaces). For continuous parameter martingales with continuous paths, this definition was introduced by Bañuelos and Wang [BW1] and for continuous parameter martingales by Wang [W]. With this definition of subordination, Bañuelos and Wang [BW1] and Wang [W] extended various sharp martingale inequalities of Burkholder [Bu1–Bu5] from the discrete-time and certain stochastic integral settings to general continuous parameter martingales. In particular, the following theorem was proved in Wang [W] (see also [BW1]). We use the notation $\|X\|_p$ to denote $\sup_{t\geq 0}\|X_t\|_p$.

Theorem 1.1. *Let X and Y be two $\mathbb{H}$-valued continuous-time parameter martingales such that Y is differentially subordinate to X. Then, for $1<p<\infty$,*

$$\|Y\|_p \leq (p^*-1)\|X\|_p. \tag{1.1}$$

This inequality is sharp, and it is also strict if $p\neq 2$ and $0<\|X\|_p<\infty$.

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