Michigan Math. J. 47 (2000)

Dimension of Julia Sets of Polynomial Automorphisms of $\mathbf{C}^2$

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1. Introduction

Let g be a polynomial automorphism of $\mathbf{C}^2$. In a similar way as is done for polynomials in $\mathbf{C}$, we denote by $K^{\pm}$ the set of points in $\mathbf{C}^2$ with bounded forward/backward orbit under g. We write $J^{\pm} = \partial K^{\pm}$ and $J = J^{+} \cap J^{-}$. We refer to $J^{\pm}$ as the positive/negative Julia set and to J as the Julia set of g. The set $J^{\pm}$ is unbounded, closed, and connected, while J is compact (see [BS2; BS3; FM; HO] for more details).

The purpose of the main part of this paper is to show that, under the assumption that g is a hyperbolic mapping (i.e., the Julia set J is a hyperbolic set for g), the complete information about the Hausdorff dimensions of J^{+} and J^{-} is already contained in the Julia set J itself. In particular, the results of Theorem 4.1–4.4 can be summarized by the following result.

Theorem 1.1. *Let g be a hyperbolic polynomial automorphism of $\mathbf{C}^2$ and let $p \in J$. Then*

(i) $\dim_H J^{\pm} = \dim_H W_{\varepsilon}^{u/s}(p) \cap J + 2$;
(ii) $2 < \dim_H J^{\pm} < 4$;
(iii) $\dim_H J = \dim_H J^{+} + \dim_H J^{-} - 4$.

The main idea in the proof of Theorem 1.1(i) is to construct locally a lamination of $\mathbf{C}^2$ such that the intersection of its leaves with $J^{\pm}$ can be represented as the image of $W_{\varepsilon}^{u/s}(p) \cap J$ under a particular holomorphic motion. It is then possible to verify that locally the Hausdorff dimension of $J^{\pm}$ is arbitrarily close to that of $W_{\varepsilon}^{u/s}(p) \cap J + 2$.

Only partial results are known about the Hausdorff dimensions of J^{+} and J^{-} (see [FoS; Wo]). One difficulty for a direct calculation is that both J^{+} and J^{-} are unbounded sets, and every restriction of g to a sufficiently large (in the sense of Hausdorff dimension) compact subset leads—either under forward or under backward iteration—out of the set. On the other hand, a result of Verjovsky and Wu [VW] shows that the Hausdorff dimension of $W_{\varepsilon}^{u/s}(p) \cap J$ can be calculated in terms of Bowen's formula. Therefore, Theorem 1.1(i) relates the Hausdorff dimension of $J^{\pm}$ to Bowen's formula.

Received May 9, 2000. Revision received September 20, 2000.
The author was supported by a Forschungsstipendium of the Deutsche Forschungsgemeinschaft (DFG).