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A Complete Bounded Minimal Cylinder in $\mathbb{R}^3$

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1. Introduction

Calabi asked if it were possible to have a complete minimal surface in $\mathbb{R}^3$ entirely contained in a half-space. As a consequence of the strong half-space theorem [6], no such surfaces are properly immersed. The first examples of complete orientable nonflat minimal surfaces with a bounded coordinate function were obtained by Jorge and Xavier [7]. Their construction is based on an ingenious idea of using Runge's theorem. Later, Brito [1] discovered a new method to construct surfaces of this kind. Examples of complete minimal surfaces with nontrivial topology, contained in a slab of $\mathbb{R}^3$, were obtained by Rosenberg and Toubiana [12], López [8; 9], Costa and Simoes [3], and Brito [2], among others.

A few years ago, Nadirashvili [10] used Runge's theorem in a more elaborate way to produce a complete minimal disc inside a ball in $\mathbb{R}^3$ (see also [4]).

In this paper we generalize the techniques used by Nadirashvili to obtain new examples of complete minimal surfaces inside a ball in $\mathbb{R}^3$ that have the conformal structure of an annulus. To be more precise, we have proved the following.

THEOREM 1. *There exist an open set A of $\mathbb{C}$ and a complete minimal immersion $X: A \to \mathbb{R}^3$ satisfying*:

(1) *$X(A)$ is a bounded set of $\mathbb{R}^3$*;

(2) *A has the conformal type of an annulus.*

This theorem is proved in Section 3.

We have obtained the immersion X as limit of a sequence of bounded minimal annuli with boundary. To construct the sequence we require a technical lemma whose proof is exhibited in Section 4. This lemma allows us to modify the intrinsic metric of a minimal annulus around the boundary without excessively increasing the diameter of the annulus in $\mathbb{R}^3$.

2. Background and Notation

The aims of this section are to establish the principal notation used in the paper and to summarize some results about minimal surfaces.

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