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Isometric Actions and Harmonic Morphisms

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Introduction

It is well known that a Riemannian foliation with minimal leaves has the property that it produces harmonic morphisms, that is, *its leaves are locally fibers of submersive harmonic morphisms.* This is an immediate consequence of the fact that Riemannian submersions with minimal fibers are harmonic morphisms.

More generally, a Riemannian foliation (of codimension not equal to 2) produces harmonic morphisms if and only if the vector field determined by the mean curvatures of the leaves is locally a gradient vector field. This is a consequence of the fundamental equation of Baird and Eells [1] (see Proposition 1.2 in the next section). Although this condition is quite simple, few examples of such Riemannian foliations were known; our work will provide many new ones.

For a 1-dimensional Riemannian foliation, the condition just stated is equivalent to the fact that the foliation is locally generated by Killing fields (a result due to Bryant [6]), but this is not true for foliations of dimension greater than 1. In this paper we show that, for a foliation locally generated by Killing fields, the condition depends only on the integrability tensor of the horizontal distribution and the induced local action. Thus we obtain a useful criterion for a foliation locally generated by Killing fields to produce harmonic morphisms. This is done in Section 1 (Theorem 1.13). In Section 2 we derive a few consequences, thus obtaining the following classes of Riemannian foliations (of codimension $\neq 2$) that produce harmonic morphisms:

(a) foliations locally generated by Killing fields and with integrable orthogonal complement;
(b) foliations generated by the local action of an abelian Lie group of isometries;
(c) foliations generated by the action of a unimodular closed subgroup of the isometry group;
(d) foliations generated by the action of a Lie group of isometries whose orbits are naturally reductive homogeneous Riemannian manifolds;
(e) foliations formed by the fibers of principal bundles for which the total space is endowed with a metric such that the structural group acts as an isometry

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