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Pseudo-Carleson Measures for Weighted Bergman Spaces

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1. Problem and Solution

Let $\triangle$ and dm be the open unit disk and the 2-dimensional Lebesgue measure on the complex plane $\mathbb{C}$, respectively. For $\alpha \in (-1, \infty)$, put $dm_\alpha(z) = \pi^{-1}(\alpha+1)(1-|z|^2)^\alpha\, dm(z)$. For $p \in [1, \infty)$, let A^p_α denote the weighted Bergman space of all analytic functions f on $\triangle$ for which

$$\|f\|^p_{A^p_\alpha} = \int_\triangle |f|^p\, dm_\alpha < \infty.$$

This definition breaks down at $p = \infty$. The space A^∞_α is substituted by the Bloch space $\mathcal{B}$, which consists of those analytic functions f on $\triangle$ obeying

$$\|f\|_{\mathcal{B}} = |f(0)| + \sup_{z\in\triangle}(1-|z|^2)|f'(z)| < \infty.$$

Every function $f \in A^1_\alpha$ has the reproducing formula [10, p. 53]

$$f(z) = \int_\triangle \frac{f(w)}{(1-\bar{w}z)^{\alpha+2}}\, dm_\alpha(w), \quad z \in \triangle.$$

Note that A^p_α decreases with p and has the duality properties (cf. [3, Thm. 2.4, Thm. 2.5]) $[A^p_\alpha]^* \cong A^q_\alpha$ for $p > 1$ and $p^{-1} + q^{-1} = 1$; whereas $[A^1_\alpha]^* \cong \mathcal{B}$ under the pairing

$$\langle f, g\rangle_\alpha = \int_\triangle f\bar{g}\, dm_\alpha.$$

A word of caution is necessary: the last integral is understood in the sense of conditional convergence,

$$\lim_{r\to 1^-} \int_{|z|\le r} f(z)\overline{g(z)}\, dm_\alpha(z),$$

rather than absolute convergence (which, in fact, is false in some cases).

After giving a lecture (about Möbius invariant function spaces) on March 23, 1998, in the Department of Mathematics of Lund University, Sweden, I was encouraged by J. Peetre to attack the following problem.

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