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On the Coverings of Proper Families of 1-Dimensional Complex Spaces

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1. Introduction

In this article we want to show the following result concerning the stability of holomorphic convexity for covering spaces.

Theorem 1.1. *Let $\pi : X \to T$ be a proper holomorphic surjective map of complex spaces, let $t_0 \in T$ be any point, and denote by $X_{t_0} := \pi^{-1}(t_0)$ the fiber of π at t_0. Assume that* $\dim X_{t_0} = 1$. *Let $\sigma : \tilde{X} \to X$ be a covering space and let $\tilde{X}_{t_0} = \sigma^{-1}(X_{t_0})$. If $\tilde{X}_{t_0}$ is holomorphically convex, then there is an open neighborhood D_1 of t_0 such that $(\pi \circ \sigma)^{-1}(D_1)$ is holomorphically convex.*

Remark 1.2. This result is the main achievement of the note of T. Ohsawa [8]. However, as will be explained at the end of our article, we have serious questions about his proof. Therefore, we consider it necessary to give a complete and clear proof of Theorem 1.1.

Our theorem will be a consequence of the following proposition.

Proposition 1.3. *Let $\pi : X \to T$ be a proper holomorphic surjective map of complex spaces, let $t_0 \in T$ be any point, and denote by $X_{t_0} := \pi^{-1}(t_0)$ the fiber of π at t_0. Assume that* $\dim X_{t_0} = 1$. *Let $\sigma : \tilde{X} \to X$ be a covering space and let $\tilde{X}_{t_0} := \sigma^{-1}(X_{t_0})$. If $\tilde{X}_{t_0}$ is holomorphically convex, then there exist*:

(1) *an open neighborhood D of t_0*;
(2) *a continuous plurisubharmonic vertical exhaustion function*

$$f : \tilde{D} := (\pi \circ \sigma)^{-1}(D) \to \mathbb{R}_+$$

(*i.e., the restriction of $\pi \circ \sigma : \tilde{D} \to D$ to $\{f \le c\}$ is proper for every $c \in \mathbb{R}$*); *and*

(3) *an increasing sequence $\{a_\nu\}$, $a_\nu \to \infty$, such that f is strongly plurisubharmonic near the level sets $\{f = a_\nu\}$, $\nu \in \mathbb{N}$.*

Remark 1.4. This proposition is proved by Napier [5] for $\dim X = 2$, $\dim T = 1$, and X, T smooth.

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