Michigan Math. J. 47 (2000)

Artin Groups of Finite Type with Three Generators

Thomas Brady

1. Introduction

Let W be a finite Coxeter group on three generators A, B, C, and consider the set of all possible expressions of the Coxeter element $X = BAC$ in W as a product of three reflections. In Section 3 we will construct a 3-dimensional CW-complex, $K(W)$, which we can associate to this set in a natural way. (Daan Krammer has informed us that he has also considered this complex and has obtained similar results.) We will show that this complex enjoys two remarkable properties. First, the fundamental group of $K(W)$ is isomorphic to the finite type Artin group determined by W; second, $K(W)$ can be given a piecewise Euclidean (PE) metric of nonpositive curvature. Thus, if G is an Artin group of finite type with three generators, then G acts cocompactly on a 3-dimensional PE complex of nonpositive curvature.

The paper is arranged as follows. In Section 2, we make the corresponding construction for two-generator Artin groups and prove that the associated 2-complex has nonpositive curvature. In Section 3, we define the complex K and show that it has the correct fundamental group. In Section 4, we give K a PE metric and show that it has nonpositive curvature.

We would like to thank the Mathematics Department at Brigham Young University, where most of this work was completed.

2. Artin Groups of Finite Type with Two Generators

Let W be a finite Coxeter group on two generators A and B; that is, $W = W_m$ has a presentation of the form

$$W_m = \langle A, B \mid A^2 = B^2 = (AB)^m = 1\rangle.$$

Thus W_m is the dihedral group of order $2m$, which can be thought of as a finite reflection group acting on $\mathbf{R}^2$. The elements A and B act as reflections in lines through the origin that make an angle of π/m while the element AB acts by rotation through $2\pi/m$. The corresponding Artin group is the group with the presentation

$$G_m = \langle a, b \mid \text{prod}(a, b; m) = \text{prod}(b, a; m)\rangle,$$

Received September 7, 1999. Revision received March 7, 2000.