Gluck Surgery along a 2-Sphere in a 4-Manifold is Realized by Surgery along a Projective Plane

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0. Introduction

One of the well-known methods to construct a new 4-manifold from an old one is the Gluck surgery along an embedded 2-sphere with trivial normal bundle, which is defined as follows (see [G]). Let M be a smooth 4-manifold and K a smoothly embedded 2-sphere in M. We suppose that the tubular neighborhood $N(K)$ of K in M is diffeomorphic to $S^2 \times D^2$. Let τ be the self-diffeomorphism of $S^2 \times S^1 = \partial(S^2 \times D^2)$ defined by $\tau(z, \alpha) = (\alpha z, \alpha)$, where we identify S^1 with the unit circle of $\mathbf{C}$ and S^2 with the Riemann sphere $\hat{\mathbf{C}} = \mathbf{C} \cup \{\infty\}$. Then consider the 4-manifold obtained from $M - \operatorname{Int} N(K)$ by regluing $S^2 \times D^2$ along the boundary using τ. We say that the resulting 4-manifold, denoted by $\Sigma(K)$, is obtained from M by the *Gluck surgery* along K (see [G] or [Kir2, p. 16]).

When the ambient 4-manifold is the 4-sphere S^4, we call a smoothly embedded 2-sphere K in S^4 a 2-*knot*. In this case, the resulting 4-manifold $\Sigma(K)$ is always a homotopy 4-sphere. It has been known that, for certain 2-knots K, $\Sigma(K)$ is again diffeomorphic to S^4 (see e.g. [Gom1; Gor; HMY; Mo; Pl]). It has not been known if the Gluck surgery along a 2-knot K in S^4 produces a 4-manifold $\Sigma(K)$ not diffeomorphic to S^4 for some K (see [Kir1, 4.11, 4.24, 4.45] and [Gom2]). On the other hand, for 2-spheres embedded in 4-manifolds M not necessarily diffeomorphic to S^4, Akbulut [Ak1; Ak2] constructed an example of an embedded 2-sphere K in such an M such that $\Sigma(K)$ is homeomorphic but is not diffeomorphic to M. For Gluck surgeries, see also [Ak3; AK; AR; Gom1; Gom2].

Price [P] considered a similar construction using embedded projective planes in S^4. Let P be a smoothly embedded projective plane in S^4. In the following, we fix an orientation for S^4. Then it is known that the tubular neighborhood $N(P)$ of P is always diffeomorphic to the nonorientable D^2-bundle over $\mathbf{R}P^2$ with Euler number ± 2 (see [M1; M2]), which we denote by N_e with $e = \pm 2$ the Euler number. Note that ∂N_e is diffeomorphic to the quaternion space Q, whose fundamental

Received December 18, 1998. Revision received June 22, 1999.
The second author has been partially supported by the Anglo-Japanese Scientific Exchange Programme, run by JSPS and the Royal Society. The fourth author was partially supported by JSPS Research Fellowship for Young Scientists.
Michigan Math. J. 46 (1999).