Isometric Cusps in Hyperbolic 3-Manifolds

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1. Introduction

Given a cusped finite-volume hyperbolic 3-manifold M, each cusp has two natural invariants associated to it. First, there is the so-called cusp shape, which is the Euclidean metric on a torus cross-section of the cusp, up to scaling. It can be described by an equivalence class of parallelograms that correspond to fundamental domains for the action of the cusp subgroup on a horizontal plane in the upper half-plane model. The second natural invariant is the maximal cusp volume, which is obtained by expanding the cusp until it first touches itself and then computing the corresponding volume.

In [10] it was proved that the set of possible cusp shapes corresponding to cusps in hyperbolic 3-manifolds is dense in the set of possible Euclidean metrics on a torus. In some sense, one expects a panoply of nonisometric cusps in different manifolds.

However, in [8] and [9], the authors provided examples of manifolds (and orbifolds) with two cusps such that surgery on one cusp leaves the cusp shape of the remaining cusp invariant. In particular, this generates an infinite set of manifolds, each with a single cusp having the same cusp shape. In [3] it was demonstrated that these examples also have the same maximal cusp volume.

Define two cusps in two possibly distinct hyperbolic 3-manifolds to be *maximally isometric* if there is an isometry of the interior of one maximal cusp to the interior of the other. In particular, this occurs if and only if both the cusp shape and the maximal cusp volume are the same for the two cusps.

In this paper, a list of “generic cusps” is provided and defined up to maximal isometry such that one can choose one of these cusps and then—by removing three disjoint simple closed curves from any closed 3-manifold or two disjoint simple closed curves from any cusped hyperbolic 3-manifold—the resulting manifold is hyperbolic and one of the new cusps is maximally isometric to the chosen cusp. The set of generic cusps contains a large variety of cusps, including ones of maximal cusp volume 4, 6, $2\sqrt{3}$, $2(1+\sqrt{3})$, and $8\sqrt{3/5}$. Moreover, if one removes four disjoint simple closed curves from a closed hyperbolic 3-manifold, or three

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