Residual Amenability and the Approximation of L^2-Invariants

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Introduction

In 1994, Wolfgang Lück [15] proved the beautiful theorem that, if X is a finite simplicial complex with residually finite fundamental group, then the L^2-Betti numbers of the universal covering of X can be approximated by the ordinary Betti numbers of a sequence of finite coverings of X. In fact, the question of approximation dates back to Kazhdan [12] (see also [10, p. 231]), but only an inequality was known. Dodziuk and Mathai [8] show a result analogous to Lück's theorem in the situation where the covering transformation group is amenable. Specifically, they show that the L^2-Betti numbers of an amenable covering $\tilde{X}$ of X can be approximated by the ordinary Betti numbers of a sequence of Følner subsets of $\tilde{X}$. This paper generalizes Lück's theorem to the case where the cover of X has residually amenable transformation group, a large class of groups that includes the residually finite groups of Lück's theorem and the amenable groups of Dodziuk and Mathai.

In this paper we also consider L^2-torsion. At first, L^2-torsion was defined for L^2-acyclic covering spaces. The L^2-analytic torsion was first studied in [18] and [14], and L^2-Reidemeister–Franz torsion was first studied in [6] (see also [16]). Equality of the combinatorial and analytic L^2-torsions was proven in 1996 [4].

In order to define these L^2-torsions, one needs to establish decay near zero of the spectral density function for the L^2-Laplacian. In the case of a residually finite covering, Lück [15] derives an elegant estimate on the spectral density functions for the finite covers that in the limit gives the necessary decay for the combinatorial L^2-Laplacian. Lück also proves the homotopy invariance of L^2-combinatorial torsion in this case.

In [5], the combinatorial and analytic torsion invariants are defined more generally as volume forms on L^2-cohomology, with the decay condition on the spectrum now replaced by a similar condition known as determinant class. The results of [4] extend to show the equality of these more general combinatorial and analytic L^2-torsions.

Dodziuk and Mathai [8] show that coverings with amenable covering group are of determinant class. Mathai and Rothenberg [19] recently extended Lück's results to prove the homotopy invariance of L^2-torsion in that case. Although an error in

Received July 15, 1998. Revision received April 21, 1999.
Michigan Math. J. 46 (1999).