Differential Polynomials That Share Three Finite Values with Their Generating Meromorphic Function

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1. Introduction

In this paper, "meromorphic function" means meromorphic in the whole plane $\mathbb{C}$. We shall assume that the reader is familiar with the notation and elementary aspects of Nevanlinna theory (cf. [3] or [4]).

We say that two meromorphic functions f and g share a value a "IM" (resp. CM) if $f - a$ and $g - a$ have the same zeros ignoring multiplicities (counting multiplicities). The subject on sharing values between meromorphic functions and their derivatives was first studied by Rubel and Yang [9].

THEOREM A. *Let f be a nonconstant entire function. If f and f' share two finite values CM, then $f = f'$.*

This result was improved independently by Gundersen [2], and Mues and Steinmetz [7].

THEOREM B. *Let f be meromorphic and nonconstant. If f and f' share three finite and distinct values b_1, b_2, b_3 IM, then $f = f'$.*

Frank and Schwick [1] generalized this to the kth derivative.

THEOREM C. *Let f be meromorphic and nonconstant, $k \in \mathbb{N}$. If f and $f^{(k)}$ share three finite and distinct values b_1, b_2, b_3 IM, then $f = f^{(k)}$.*

In the sequel, we set

$$L(f) := a_k f^{(k)} + a_{k-1} f^{(k-1)} + \cdots + a_0 f \quad (a_k \neq 0), \tag{1}$$

where $a_k, \dots, a_0$ are finite constants. Mues-Reinders [6] proved the following result.

THEOREM D. *Let f be meromorphic and nonconstant, $2 \le k \le 50$. If f and $L(f)$ share three finite and distinct values b_1, b_2, b_3 IM, then $f = L(f)$. Furthermore, if $a_{k-1} = a_{k-2} = 0$, then the restriction $k \le 50$ can be omitted.*

Received July 9, 1998. Revision received December 7, 1998.

The second author was supported by DAAD/K.C. Wong Fellowship, NSF of China, and NSF of Jiangsu Province.

Michigan Math. J. 46 (1999).