Essential Surfaces and Tameness of Covers

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Suppose that M is a closed orientable 3-manifold. An *essential surface* S in M is a π_1-injective map of a closed surface S to M. Throughout this paper, we will restrict ourselves to the case of S being orientable. The surface cover M_S of S is said to be *topologically tame* if it is homeomorphic to $S \times R$. Equivalently, there is a compactification of the surface cover homeomorphic to $S \times [-1, 1]$. In this paper we establish tameness of surface covers for two natural classes of essential surfaces. The first class we call *topologically finite.* The defining properties of such surfaces are conditions that are easily seen to be satisfied by quasi-Fuchsian surfaces in hyperbolic 3-manifolds (cf. [RS1]). Using geometric techniques, it is straightforward to check that quasi-Fuchsian surfaces in a hyperbolic 3-manifold have topologically tame surface covers. In fact, M_S can be compactified by adding the quotient of the domain of discontinuity by the action of $f_*(\pi_1(S))$ at infinity (see e.g. [Th]). We give an alternate topological proof of tameness by using three important properties of geometrically finite surfaces. The proof is reminiscent of an argument in [HRS] that establishes this tameness for the case when S is a torus.

The second class of essential surfaces that we address are called *strongly filling. A filling surface S* is essential and satisfies the condition that every noncontractible loop in the 3-manifold always intersects every surface in the homotopy class of S. This immediately implies that the complementary regions of S are all simply connected when a least area representative surface is picked in the homotopy class of S, regardless of the choice of metric. In order to make this into a property that ensures tameness, we need a notion of strongly filling for an essential surface S. Strongly filling for S means that, in the universal cover of M, all pairs of points (that are sufficiently far apart) are separated by many disjoint planes lying over S. See Section 2. For quasi-Fuchsian surfaces in hyperbolic 3-manifolds, we show that it is sufficient to assume that every geodesic line has endpoints separated by at least one plane in the preimage of S. If S is totally geodesic in the hyperbolic case, we establish that strongly filling and filling are equivalent. Finally, we will show that if all essential surfaces are either strongly filling or topologically finite, then topological tameness is always true not only of the surface cover but also of any cover with finitely generated and freely indecomposable fundamental group. It is a well-known result of Simon [Si] that this is true for Haken 3-manifolds.

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