Bounded Operators and Isomorphisms of Cartesian Products of Fréchet Spaces

P. Djakov, T. Terzioğlu, M. Yurdakul, & V. Zahariuta

Introduction

In [25; 26] it was discovered that there exist pairs of wide classes of Köthe spaces $(\mathcal{X}, \mathcal{Y})$ such that

$$L(X, Y) = LB(X, Y) \quad \text{if } X \in \mathcal{X},\ Y \in \mathcal{Y}, \tag{1}$$

where $LB(X, Y)$ is the subspace of all bounded operators from X to Y. If either any $X \in \mathcal{X}$ is Schwartzian or any $Y \in \mathcal{Y}$ is Montel, then this relation coincides with

$$L(X, Y) = L_c(X, Y) \quad \text{if } X \in \mathcal{X},\ Y \in \mathcal{Y}, \tag{2}$$

where $L_c(X, Y)$ denotes the subspace of all compact operators.

This phenomenon was studied later by many authors (see e.g. [1; 5; 11; 12; 13; 14; 15; 20; 21]); of prime importance are Vogt's results [24] giving a generally complete description of the relations (1) for the general case of Fréchet spaces (for further generalizations see also [3; 4]).

Originally, the main goal in [25; 26] was the isomorphism of Cartesian products (and, consequently, the quasi-equivalence property for those spaces). The papers made use of the fact that, due to Fredholm operators theory, an isomorphism of spaces $X \times Y \simeq X_1 \times Y_1$ ($X, X_1 \in \mathcal{X}$, $Y, Y_1 \in \mathcal{Y}$) that satisfies (2) also implies an isomorphism of Cartesian factors "up to some finite-dimensional subspace".

In the present paper we generalize this approach onto classes $\mathcal{X} \times \mathcal{Y}$ of products that satisfy (1) instead of (2). Although Fredholm operators theory fails, we have established that—in the case of Köthe spaces—the stability of an automorphism under a bounded perturbation still takes place, but in a weakened form: "up to some basic Banach space". In particular, we get a positive answer to Question 2 in [7]: Is it possible to modify somehow the method developed in [25; 26] in order to obtain isomorphic classification of the spaces $E_0(a) \times E_\infty(b)$ in terms of sequences a, b if $a_i \not\to \infty$ and $b_i \not\to \infty$?

Some of our results are announced without proofs in [9].

Received October 16, 1997. Revision received May 28, 1998.
Research of the first author was supported by the TÜBİTAK–NATO Fellowship Program and partially by the NRF of Bulgaria, grant no. MM-808/98.
Michigan Math. J. 45 (1998).