A Canonical Differential Complex for Jacobi Manifolds

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1. Introduction

Jacobi structures were independently introduced by Lichnerowicz [27; 28] and Kirillov [21], and they are a combined generalization of symplectic or Poisson structures and of contact structures.

A Jacobi structure on an n-dimensional manifold M is a pair (Λ, E), where Λ is a skew-symmetric tensor field of type $(2,0)$ and E a vector field on M verifying $[\Lambda,\Lambda] = 2E\wedge\Lambda$ and $[E,\Lambda] = 0$. The manifold M endowed with a Jacobi structure is called a Jacobi manifold. A bracket of functions (called Jacobi bracket) is then defined by $\{f,g\} = \Lambda(df,dg) + fE(g) - gE(f)$. Thus, the algebra $C^\infty(M,\mathbb{R})$ of C^∞ functions on M, endowed with the Jacobi bracket, is a local Lie algebra in the sense of Kirillov (see [21]). Conversely, a structure of local Lie algebra on $C^\infty(M,\mathbb{R})$ defines a Jacobi structure on M (see [16; 21]). When E identically vanishes, we recover the notion of Poisson manifold. Another link between Jacobi and Poisson manifolds is the following. Take a regular Jacobi manifold, that is, the vector field E defines a regular foliation; thus, the quotient manifold inherits a Poisson structure.

The purpose of this paper is to extend to Jacobi manifolds the construction of the canonical double complex for Poisson manifolds due to Koszul [23] and Brylinski [6]. The first step is to define an appropiate differential operator $\delta = [i(\Lambda), d]$ that extends the one introduced by Koszul [23] and Brylinski [6]. The restriction of δ to the complex of basic differential forms $\Omega^*_B(M)$ is a homology operator, and the resultant homology groups will be called canonical. Motivated by Brylinski, we propose the following problem.

Problem A-J. Give conditions on a compact Jacobi manifold which ensure that any basic cohomology class in $H^*_B(M)$ has a harmonic representative α, that is, $d\alpha = 0$ and $\delta\alpha = 0$.

Moreover, the relation $\delta d + d\delta = 0$ allows us to introduce a double complex. Associated with it, there exist two spectral sequences. The second spectral sequence always degenerates at the first term; however, this is not true for the first one. Hence we propose the following problem.

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