Properly Immersed Singly Periodic Minimal Cylinders in $\mathbb{R}^3$

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1. Introduction

In 1867, Riemann [16] discovered a 1-parameter family of minimal surfaces foliated by circles and lines in parallel planes. Since then, many other mathematicians have characterized these examples from different points of view. Some of these characterizations can be seen in [1; 2; 4; 6; 14; 19].

Very recently, Meeks, Pérez, and Ros [9] have characterized the plane, the catenoid, the helicoid, and the Riemann examples as the only properly embedded genus-0 minimal surfaces with an infinite number of symmetries, and it is conjectured (see [8] and [18]) that this result remains valid without the hypothesis of an infinite number of symmetries.

In particular, the Riemann examples are the only properly embedded minimal tori with a finite number of planar ends in $\mathbb{R}^3/\mathcal{T}$, where $\mathcal{T}$ is the group generated by a nontrivial translation, which improves the aforementioned results.

Previously, Pérez and Ros [15] had proved that there are no properly embedded minimal surfaces of genus 1 and a finite number of planar ends in $\mathbb{R}^3/\mathcal{S}_\theta$, where $\mathcal{S}_\theta$ is a group generated by a screw motion of angle $\theta \neq 0$.

Observe that the Meeks–Pérez–Ros theorem can be stated by saying that any properly embedded minimal torus in $\mathbb{R}^3/\mathcal{T}$ with $2n$ ends is a covering of a torus in $\mathbb{R}^3/(\mathcal{T}/n)$ with two ends.

In this paper we study the same kind of questions in the more general immersed case: *Is a properly immersed minimal torus with $2n$ ends in $\mathbb{R}^3/\mathcal{T}$ a covering of a torus in $\mathbb{R}^3/(\mathcal{T}/n)$ with two ends?* López, Ritoré, and Wei [6] have found all complete minimal immersed tori in $\mathbb{R}^3/\mathcal{T}$ with two parallel planar embedded ends. This moduli space consists of a countable number of regular curves and, with the exception of Riemann examples, each one of these curves contains at least one point that provides a surface with vertical flux, and hence they are not embedded (see [7] and [15]).

We give an affirmative answer to the question just posed when dealing with properly immersed minimal tori with four planar ends. Toward that end, we prove the following theorem.

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