p-Groups of Symmetries of Surfaces

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1. Introduction

Let Σ_g denote a closed orientable surface of genus $g \geq 2$. Let G be a nontrivial finite group. If G can be embedded in the group of orientation-preserving self-homeomorphisms of Σ_g, then we say that G *acts on* Σ_g. In this case, Σ_g can be realized as a Riemann surface and G as a subgroup of its automorphism group.

For each fixed g, there can be only finitely many finite groups G that act on Σ_g, since by a famous result of Hurwitz [11] the order of G is bounded above by $84(g-1)$. For some small values of g, complete listings of those groups which act on Σ_g have been obtained (see e.g. [15; 16; 19; 27]).

On the other hand, for each G there is an infinite sequence of values of g such that G acts on Σ_g [13; 22]. The determination of this sequence, here called the *genus spectrum* of G, is termed the *Hurwitz problem* in [22]. The genus spectrum for a cyclic group of prime order was determined in [12; 14; 22] and can be deduced from earlier results [17; 8] (see also [5]).

Much effort has gone in to determining the smallest member of this genus spectrum for various classes of groups [3; 6; 7; 19]. Indeed, moving beyond the restrictions imposed here—that is, to consider nonclosed and/or nonorientable surfaces or allowing the self-homeomorphisms to be orientation reversing—the corresponding smallest numbers have been widely investigated (see [1] and the references there).

To describe the results of this paper, we use the notation that evolved from [13; 14] as follows: For each finite group G, there is an integer $n_0(G)$, easily computed from the Sylow subgroup structure of G, such that if G acts on Σ_g then $g = 1 + n_0(G)g_0$ for some $g_0 \geq 1$. The integer g_0 is called a *reduced genus for* G. Let $\mu_0 = \mu_0(G)$ denote the *minimum reduced genus* for G and let $\sigma_0 = \sigma_0(G)$ denote the *minimum stable reduced genus* for G, that is, minimal with the property that all $g_0 \geq \sigma_0$ are reduced genera. In addition, the integers in the interval $[\mu_0, \sigma_0]$ that do not occur as reduced genera of G will constitute the (*reduced*) *gap sequence* of G. The infinite sequence of integers

$$\{\, g \geq 2 \mid G \text{ acts on } \Sigma_g \,\}$$

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