F-Rationality of Determinantal Rings and Their Rees Rings

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Let X be an $m \times n$ matrix ($m \le n$) of indeterminates over a field K of positive characteristic, and denote the ideal generated by its t-minors by I_t. We show that the Rees ring $\mathcal{R}(I_t)$ of $K[X]$, as well as the algebra A_t generated by the t-minors, are F-rational if $\operatorname{char} K > \min(t, m-t)$. Without a restriction on characteristic this holds for $K[X]/I_{r+1}$ and the symbolic Rees ring $\mathcal{R}^s(I_t)$. The determinantal ring $K[X]/I_{r+1}$ is actually F-regular, as was previously proved by Hochster and Huneke [13] and Conca and Herzog [5] through different approaches.

Our main tool is the filtration induced by the straightening law. The associated graded ring with respect to this filtration is typically given by a Segre product $K[H] \#_{\mathbb{N}^m} F(X)$, where H is a normal semigroup representing the weights of the standard bitableaux present in the object under consideration, $\mathbb{N}^m$ represents all the possible weights, and $F(X)$ parameterizes the set of standard bitableaux of $K[X]$. The ring $F(X)$ itself is the Segre product $F_1(X) \#_{\mathbb{N}^m} F_2(X)$, where $F_1(X)$ (resp. $F_2(X)$) are the coordinate rings of the flag varieties associated with X (resp. the transpose of X).

We prove that $F(X)$ is F-regular. Normal semigroup rings are also F-regular since they are direct summands of polynomial rings. Furthermore, F-regularity is inherited by Segre products, and F-rationality is preserved under deformations. Hence a ring with an associated graded ring of type $K[H] \#_{\mathbb{N}^m} F(X)$ is (at least) F-rational. This applies especially to $K[X]/I_{r+1}$, $\mathcal{R}(I_t)$, and A_t.

The results and the method of this paper are a variant of the method applied by Bruns [1] in characteristic 0, where F-rationality is to be replaced by the property of having rational singularities. By a theorem of Smith [15], our results in positive characteristic actually imply those previously obtained in characteristic 0.

1. The Filtration Induced by the Straightening Law

In this section we discuss the filtration on $K[X]$ induced by the straightening law and identify its associated graded ring. The filtration was first described by De Concini, Eisenbud, and Procesi [7]. We use the language of Young tableaux; for unexplained terminology the reader is referred to Bruns and Vetter [4, Sec. 11].

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