Maximal Gleason Parts for H^∞

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Introduction

It is well known that every Gleason part of the algebra H^∞, of bounded analytic functions on the unit disk, is a maximal analytic disk or a single point [7]. Furthermore, there are very different behaviors within the class of nontrivial Gleason parts. For example, it is known that some analytic disks are homeomorphic to the unit disk $\mathbb{D}$, while some others are not.

Although the Gleason parts have been studied by several authors (see e.g. [2; 5; 6]), the information at our disposal is partial and fragmented. In particular, our knowledge of the closures of Gleason parts is very limited. Far from giving the whole picture, which is probably unreachable, the present paper intends to throw some light on the behavior of the closures of Gleason parts.

First we give a criterion to check whether a point in the maximal ideal space of H^∞ is or is not in the closure of a given Gleason part. This criterion is then used to prove that if the closures of two Gleason parts have nonvoid intersection, then one of them is contained in the closure of the other. This answers a question posed by Gorkin in [5] and is the starting point of our study of maximal parts (not contained into the closure of any other part except the disk $\mathbb{D}$). We consider a class of maximal parts that contains properly the thin parts (this generalizes a result of Budde [2]) and we study the general properties of this class. Next we prove the existence of maximal parts not belonging to this class. Finally, we pose three open problems that we believe are fundamental to understanding the way in which the Gleason parts relate to each other.

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1. Preliminaries

The maximal ideal space of H^∞ is defined by

$$M(H^\infty) = \{\varphi : \varphi \text{ is linear, multiplicative and } \varphi \neq 0\}$$

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