# $E^3$-Complete Timelike Surfaces in $E_1^3$ Are Globally Hyperbolic

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## 1. Introduction

A $C^\infty$ immersion $\mathcal{Z}$ of a surface $S$ into Minkowski 3-space $E_1^3$ can also be viewed as a $C^\infty$ immersion of $S$ into Euclidean 3-space $E^3$. If the metric $I$ induced on $S$ by $\mathcal{Z}: S \to E_1^3$ is Lorentzian, then $\mathcal{Z}: S \to E_1^3$ is called *timelike*. If the metric $I_\varepsilon$ induced on $S$ by $\mathcal{Z}: S \to E^3$ is complete, then $\mathcal{Z}: S \to E_1^3$ is said to be *$E^3$-complete.*

In all results below, $S$ is a surface provided with the Lorentzian metric $I$ and the time orientation induced by a timelike $C^\infty$ immersion $\mathcal{Z}: S \to E_1^3$. Among the conformally invariant properties definable on a time-oriented Lorentzian surface are the causality conditions of interest in general relativity. Two of these conditions, stable causality and global hyperbolicity (defined in [1, pp. 63–65]) are dealt with in this paper.

Theorem 1 makes the elementary observation that $S$ must be stably causal. In case $\mathcal{Z}$ is $E^3$-complete, Theorem 2 states that $S$ is globally hyperbolic. If $\mathcal{Z}: S \to E_1^3$ is $E^3$-complete from a simply connected $S$, Theorem 3 states that $S$ is $C^\infty$-conformally diffeomorphic to a subset of the Minkowski 2-plane $E_1^2$, and places strong restrictions on the conformal boundary $\partial_0 S$, which was defined by Kulkarni in [3] and studied in [7]. (This strengthens slightly a result announced without proof on p. 196 in [7].)

Under the hypotheses of Theorem 3, Theorem 4 states that $S$ is $C^\infty$-conformally diffeomorphic to $E_1^2$ provided that the mean curvature $H$ for $\mathcal{Z}: S \to E_1^3$ vanishes outside a compact set on $S$. This generalizes the conformal Bernstein theorem from [4], which states that any entire timelike minimal surface in $E_1^3$ is $C^\infty$-conformally diffeomorphic to $E_1^2$.

Because global hyperbolicity is the most restrictive of the causality conditions on space-times discussed in [1], it may appear that the surface $S$ in Theorems 2 and 3 has little room for variation in its global Lorentzian structure. To show that this need not be the case, we describe in Section 4 uncountably many $C^0$- (and thereby $C^\infty$-) conformally distinct simply connected, globally hyperbolic subsurfaces $S_r$ of $E_1^2$. Like all subsurfaces of $E_1^2$, the $S_r$ are $C^\infty$ isometrically imbedded in $E_1^3$

Received September 11, 1996.
Both authors were partially supported under NSF grant DMS 94-01825.
Michigan Math. J. 44 (1997).