Existence of Multiple Refinable Distributions

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1. Introduction and Main Results

Wavelets and subdivision schemes are based on refinement equations of the form

$$\phi = \sum_{\alpha \in \mathbb{Z}^s} a(\alpha)\phi(2 \cdot -\alpha), \tag{1.1}$$

where $\{a(\alpha)\}_{\alpha \in \mathbb{Z}^s}$ is a sequence of complex numbers. If a is finitely supported and $\sum_{\alpha \in \mathbb{Z}^s} a(\alpha) = 2^s$, then the refinement equation (1.1) has a unique solution ϕ of compactly supported distribution on $\mathbb{R}^s$ subject to the normalized condition $\hat{\phi}(0) = 1$. Here $\hat{\phi}$ is the Fourier transform of ϕ, which is defined for an integrable function f on $\mathbb{R}^s$ by

$$\hat{f}(\xi) = \int_{\mathbb{R}^s} f(x)e^{-i\xi \cdot x}\,dx, \quad \xi \in \mathbb{R}^s,$$

and has a natural extension to compactly supported distributions. This fact of existence was proved by Cavaretta, Dahmen, and Micchelli in [1]; see also [4] and [5].

In this paper we investigate the existence of multiple refinable distributions in multiwavelets. The theory of multiwavelets began with the work of Goodman, Lee, and Tang [7; 8] and the orthogonal multiwavelet basis construction of Donovan, Geronimo, Hardin, Kessler, and Massopust [6; 9]. There have been many discussions concerning different aspects (see e.g. [2; 11; 14; 15]). All these are based on multiple refinable distributions or functions. Given a positive integer r, called the *multiplicity,* and a sequence $a := \{a(\alpha)\}_{\alpha \in \mathbb{Z}^s}$ of $r \times r$ complex matrices, a vector $\phi := (\phi_1, \dots, \phi_r)^T$ of distributions on $\mathbb{R}^s$ is called a *multiple refinable distribution* associated with the *refinement mask a* if it satisfies the following *matrix refinement equation*

$$\phi = \sum_{\alpha \in \mathbb{Z}^s} a(\alpha)\phi(2 \cdot -\alpha). \tag{1.2}$$

Note that the scalar refinement equation (1.1) is the special form of (1.2) with $r = 1$. We denote $\hat{\phi} = (\hat{\phi}_1, \dots, \hat{\phi}_r)^T$ and always set

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